Answer on Question #44560 – Math - Algebra.

If three times, the larger of the two numbers is divided by the smaller one, we get 4 as quotient and 3 as remainder. Also, if seven times the smaller number is divided by the larger one, we get 5 as quotient and 1 as remainder. Find the numbers.

Solution:

To solve given task we have consider the definition of the remainder and quotient. In algebra, the remainder is the polynomial which is "left over" after dividing one polynomial by another.

If when dividing A by B, we get a quotient Q and remainder R, then by definition, A = BQ + R (B fits into A, Q times with R left over). Euclidean division of polynomials is very similar to Euclidean division of integers and leads to polynomial remainders. Its existence is based on the following theorem: Given two univariate polynomials a(x) and b(x) (with b(x) not the zero polynomial) defined over a field (in particular, the reals or complex numbers), there exist two polynomials q(x) (the quotient) and r(x) (the remainder) which satisfy:

$$\mathrm{a(x)} = \mathrm{b(x)q(x)} + \mathrm{r(x)}$$

Where $\mathrm{q(x)}$ is the quotient and $\mathrm{r(x)}$ is the remainder, $\mathrm{a(x)}$ is the dividend and $\mathrm{b(x)}$ is divisor.

Based on the above information and condition of the task we can construct the system of equations.

Let x be the smallest number and y will be the larger one.

Then for the first condition of the task it will represent the following. The three times, the larger number is divided by the smaller one.

$$3\mathrm{y} = 4\mathrm{x} + 3$$

We know that quotient is equal to 4 and remainder is equal to 3.

We also can rewrite the equation.

$$4\mathrm{x} - 3\mathrm{y} = -3$$

The same method we apply with the second condition of the given task.

We have the quotient is equal to 5 and remainder is equal to 1. The seven times smaller number is divided by the larger one. So, we can note the following.

$$7\mathrm{x} = 5\mathrm{y} + 1$$

We can also rewrite the equation.

$$7\mathrm{x} - 5\mathrm{y} = 1$$

Now we can construct the system of the linear equations.

$$\begin{cases} 7x - 5y = 1 \\ 4x - 3y = -3 \end{cases}$$

To solve obtained system of equations we can apply the elimination method of solution by addition. We multiply the first equation by -3 and the second equation by 5.

Now we obtained the following system of equations.

$$-21x + 15y = -3$$

$$20x - 15y = -15$$

So, we can add those equations.

$$-21x + 20x + 15y - 15y = -3 + (-15)$$

Simplify the equation by combining like terms on the left and right sides.

$$-x = -18$$

Divide both sides of the equation by -1.

$$x = 18$$

Now we can find the value of y. We substitute the value of x into the first equation or in the second. It's doesn't matter in which we choose. For example, substitute x in the first equation.

$$3y = 4(18) + 3$$

Simplify the expression.

$$3y = 72 + 3$$

$$3y = 75$$

Divide both sides of the equation by 3.

$$y = 25$$

Finally we got the result. Now we have to check the obtained solution. Firstly substitute the find values into the original system of equations.

$$\begin{cases} 7(18) - 5(25) = 1 \\ 4(18) - 3(25) = -3 \end{cases}$$

Simplify by opening parentheses.

$$\begin{cases} 126 - 125 = 1 \\ 72 - 75 = -3 \end{cases}$$

$$1 = 1$$

$$-3 = -3$$

We got the true statement.

Now we have to check the condition of the given task. We find that the smaller number is equal to 18 and the lager is 25. Accordingly to the first case we know that the three times, the larger number is divided by the smaller one and it gives quotient is equal to 4 and remainder is equal to 3.

Substitute the three times larger number $25 \times 3 = 75$ then we divide by smaller one 18.

75	18
72	4
3	

We find the remainder which is equal to 3 and quotient which is equal to 4.

Now we have to check another case. The seven times smaller number is divided by the larger one and it gives quotient is equal to 5 and remainder is equal to 1.

Substitute the seven times smaller number $18 \times 7 = 126$ then we divide by larger one 25.

126	25
125	5
1	

We find the remainder which is equal to 1 and quotient which is equal to 5.

Answer: The larger number is equal to 25 and the smaller number is equal to 18.