Answer on Question #44550 – Math - Calculus

A fixed Circle C1 with equation $(x-1)^2 + y^2 = 1$ and a shrinking circle C2 with radius r and center the origin. P is the point (0,r), Q is the upper point of intersection of the two circles, and R is the point of intersection of the line PQ and the x-axis. what happense to R as C2 shrinks, that is, as r-->0^+?

Answer.

To find x-coordinate of point Q we should solve the equation:

 $1 - (1 - x)^2 = r^2 - x^2 \rightarrow 2x = r^2 \rightarrow x = \frac{r^2}{2}.$ x-coordinate of point Q: $y = \sqrt{r^2 - \left(\frac{r^2}{2}\right)^2} = \frac{r}{2}\sqrt{4 - r^2}.$

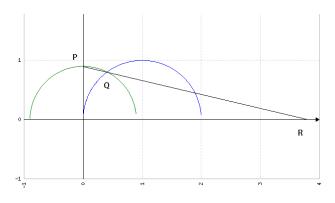
So, the equation of line through points P(0,r) and $Q(\frac{r^2}{2}, \frac{r}{2}\sqrt{4-r^2})$:

$$y-r = \frac{r-\frac{r}{2}\sqrt{4-r^2}}{0-\frac{r^2}{2}}(x-0) \rightarrow y = -\frac{2-\sqrt{4-r^2}}{r}x+r.$$

The point of intersection of our line and x-axis: $\mathbf{0} = -\frac{2-\sqrt{4-r^2}}{r} x + r \rightarrow x = \frac{r^2}{2-\sqrt{4-r^2}}$

When
$$r \to 0^+$$
, $x = \lim_{r \to 0^+} \frac{r^2}{2 - \sqrt{4 - r^2}} = \lim_{r \to 0^+} \frac{r^2}{2 - 2(1 - \frac{r^2}{8})} = 4.$

I.e., when $r \to 0^+ R \to 4$.



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