Answer on Question #44550 – Math - Calculus

A fixed Circle C1 with equation (x-1)^2 + y^2 = 1 and a shrinking circle C2 with radius r and center the origin. P is the point (0,r), Q is the upper point of intersection of the two circles, and R is the point of intersection of the line PQ and the x-axis. what happense to R as C2 shrinks, that is, as r-->0^+?

Answer.

To find x-coordinate of point Q we should solve the equation:

$$1-(1-x)^2=r^2-x^2 \ \rightarrow 2x=r^2 \rightarrow x=\frac{r^2}{2}.$$

x-coordinate of point Q: $y=\sqrt{r^2-\left(\frac{r^2}{2}\right)^2}=\frac{r}{2}\sqrt{4-r^2}.$

So, the equation of line through points $P(0,r)$ **and** $Q(\frac{r^2}{2},\frac{r}{2}\sqrt{4-r^2})$:

$$y-r=\frac{r-\frac{r}{2}\sqrt{4-r^2}}{0-\frac{r^2}{2}}(x-0) \quad \rightarrow \quad y=-\frac{2-\sqrt{4-r^2}}{r}\ x+r.$$

The point of intersection of our line and x-axis: $0=-\frac{2-\sqrt{4-r^2}}{r}\ x+r \ \rightarrow \quad x=\frac{r^2}{2-\sqrt{4-r^2}}$

When $r\rightarrow 0^+$, $x=\lim_{r\rightarrow 0^+}\frac{r^2}{2-\sqrt{4-r^2}}=\lim_{r\rightarrow 0^+}\frac{r^2}{2-2(1-\frac{r^2}{8})}=4.$

I.e., when $r\rightarrow 0^+$ $R\rightarrow 4.$