Answer on Question #44546 – Math - Algebra

Problem.

Prove that if A and B are any two sets such that A ⊆ B, then A ∪ B = B
i) by direct method;
ii) by proving its contrapositive;
iii) by contradiction.

Solution.

i) If $x \in A \cup B$, then $x \in A$ or $x \in B$. Hence $x \in B$, as $A \subseteq B$. Therefore $A \cup B \subseteq B$.
If $x \in B$, then $x \in A \cup B$ (because A ⊆ B). Therefore $B \subseteq A \cup B$.
Since $A \cup B = B$, as $A \cup B \subseteq B$ and $B \subseteq A \cup B$.

ii) We need to prove that if $A \cup B \neq B$, then $A \nsubseteq B$.
If $A \cup B \neq B$, then there exists $x \in A$ such that $x \notin B$. Therefore $A \nsubseteq B$.

iii) Suppose that $A \subseteq B$ and $A \cup B \neq B$. If $A \cup B \neq B$, then there exists $x \in A$ such that $x \notin B$. Therefore $A \nsubseteq B$. We obtain a contradiction with assumption $A \subseteq B$.