Answer on Question #44544 – Math - Linear Algebra

Problem

a) Obtain the solution set of the system x – 3y + 4z = 9, 4x + 3y + 2z = 7, y – 2x = 5 – 10z by elimination.

b) Express the following problem situation as a linear system, and then solve it by substitution. Also
show the linear system geometrically.

A manufacturer produces two types of cupboards – deluxe and regular. Each deluxe cupboard requires 12 worker hours to cut and assemble, and 5 worker hours to finish. Each regular cupboard requires 8 worker hours to cut and assemble, and 3 hours to finish. On a daily basis, the manufacturer has available 440 worker hours for cutting and assembling, and 175 worker hours for finishing. How many cupboards of each type should be produced so that all the work power is utilized?

c) Write the systems obtained in a) and (b) in matrix form.

Solution

a) $\begin{cases} x-3y+4z=9, \\ 4x+3y+2z=7, \\ y-2x=5-10z; \end{cases}$

$\begin{cases} x-3y+4z=9, \\ 4x+3y+2z=7, \\ -2x+y+10z=5; \end{cases}$ add (1) and (2):

$\left.\begin{cases} 5x+6z=16, \\ 4x+3y+2z=7, \\ -2x+y+10z=5; \end{cases}\right| \cdot(-3)$ add (2) and (3):

$\left.\begin{cases} 5x+6z=16, \\ 10x-28z=-8, \\ 6x-3y-30z=-15; \end{cases}\right| \cdot\left(-\frac{1}{2}\right)$ add (1) and (2):

$$\begin{cases} 20z=20, \\ -5x+14z=4, \\ 6x-3y-30z=-15; \end{cases}$$

$$\begin{cases} z=1, \\ -5x+14=4, \\ 6x-3y-30=-15; \end{cases}$$

$$\begin{cases} z=1, \\ x=2, \\ 12-3y=15; \end{cases}$$

$$\begin{cases} z=1, \\ x=2, \\ y=-1. \end{cases}$$

Answer: $(2;-1;1)$.

b) Assume that there should be produced x deluxe cupboards and y regular ones.
Then we have the system of equations:

$$\begin{cases} 12x+8y=440, \\ 5x+3y=175; \end{cases}$$

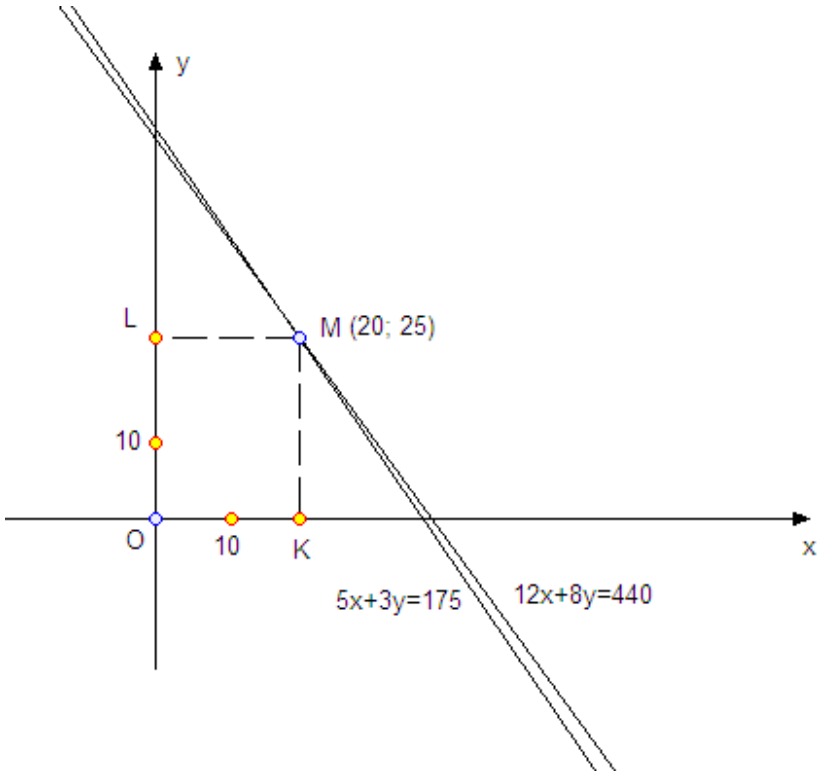
$$\begin{cases} 3x+2y=110, \\ 5x+3y=175; \end{cases}$$

$$\begin{cases} y = 55 - \frac{3}{2}x, \\ 5x + (165 - \frac{9}{2}x) = 175; \end{cases}$$

$$\begin{cases} y = 55 - \frac{3}{2}x, \\ \frac{x}{2} = 10; \end{cases}$$

$$\begin{cases} x = 20, \\ y = 25. \end{cases}$$

So, there should be produced 20 deluxe and 25 regular cupboards.



c) 1) $\begin{pmatrix} 1 & -3 & 4 \\ 4 & 3 & 2 \\ -2 & 1 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 7 \\ 5 \end{pmatrix}$

2) $\begin{pmatrix} 12 & 8 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 440 \\ 175 \end{pmatrix}$