Answer on Question #44542 – Math - Algebra

Solve the equation x^3+ 2x^2- 2x -1 = 0. using Cardano's method.

Answer.

$x^3 + 2x^2 - 2x - 1 = 0$

Substitute $x = y - \frac{2}{3} \rightarrow y^3 - \frac{10}{3}y + \frac{25}{27} = 0.$

Discriminant $\frac{1}{4}(\frac{25}{27})^2 - \frac{1}{27}\left(\frac{10}{3}\right)^3 = -\frac{3375}{2916} < 0$ **so this equation has three real end unequal roots.**

According to Cardano's formula:

$y_1 = S + T,$

$$y_2 = -\frac{S+T}{2} + \frac{\sqrt{3}}{2}(S - T)i$$

$$y_3 = -\frac{S+T}{2} - \frac{\sqrt{3}}{2}(S - T)i$$

And

$$x_1 = S + T - \frac{2}{3},$$

$$x_2 = -\frac{S+T}{2} - \frac{2}{3} + \frac{\sqrt{3}}{2}(S - T)i$$

$$x_3 = -\frac{S+T}{2} - \frac{2}{3} - \frac{\sqrt{3}}{2}(S - T)i$$

Where $S = \frac{1}{3}\sqrt[3]{-\frac{5}{2}(5 - 3\sqrt{15}i)},\ \ T = \frac{1}{3}\sqrt[3]{-\frac{5}{2}(5 + 3\sqrt{15}i)}$

As we mentioned, all the roots are real. However, they are expressed in terms of complex numbers. It can be proved that the roots cannot be expressed in terms of real radicals. This is called the irreducible case.