Answer on Question #44538 - Math - Other

a) Solve the equation $9x^4 - 18x^3 - 31x^2 + 8x + 12 = 0$ by Ferrari's method.

We have an equation $ax^4 + bx^3 + cx^2 + dx + e = 0$
where a = 9, b=-18, c=-31, d=8, e=12.

$$alpha = \frac{c}{a} - \frac{3b^2}{8a^2} = -4.944444$$

$$beta = \frac{b^3}{8a^3} - \frac{bc}{2a^2} + \frac{d}{a} = -3.555556$$

$$gamma = \frac{-3b^4}{256a^4} + \frac{cb^2}{16a^3} - \frac{bd}{4a^2} + \frac{e}{a} = 0.7291667$$

$$P = \frac{-alpha^2}{12} - gamma = -2.766461$$

$$Q = \frac{-alpha^3}{108} + alpha\frac{gamma}{3} - \frac{beta^2}{8} = -1.662767$$

$$R_p = \frac{Q}{2} + \sqrt{\frac{Q^2}{4} + \frac{P^3}{27}} = -0.8313837 + 0.3049106i$$

$$R_m = \frac{Q}{2} - \sqrt{\frac{Q^2}{4} + \frac{P^3}{27}} = -0.8313837 - 0.3049106i$$

$$U = R_m^{\left(\frac{1}{3}\right)} = 0.5740741 - 0.7698004i$$

$$y = -5\frac{alpha}{6} - U + \frac{P}{3U} = 2.972222$$

$$W = \sqrt{alpha + 2y} = 1$$

The roots are:

$$x_1 = \frac{-b}{4a} + \frac{W - \sqrt{3alpha + 2y + 2\frac{beta}{W}}}{2}$$

$$x_2 = \frac{-b}{4a} + \frac{-W + \sqrt{3alpha + 2y + 2\frac{beta}{W}}}{2}$$

$$x_3 = \frac{-b}{4a} + \frac{W - \sqrt{3alpha + 2y + 2\frac{beta}{W}}}{2}$$

$$x_4 = \frac{-b}{4a} + \frac{-W - \sqrt{3alpha + 2y + 2\frac{beta}{W}}}{2}$$

$$x_1 = 3, x_2 = \frac{2}{3}, x_3 = -1, x_4 = -\frac{2}{3}$$