Answer on Question #44537 – Math - Complex Analysis

Problem.

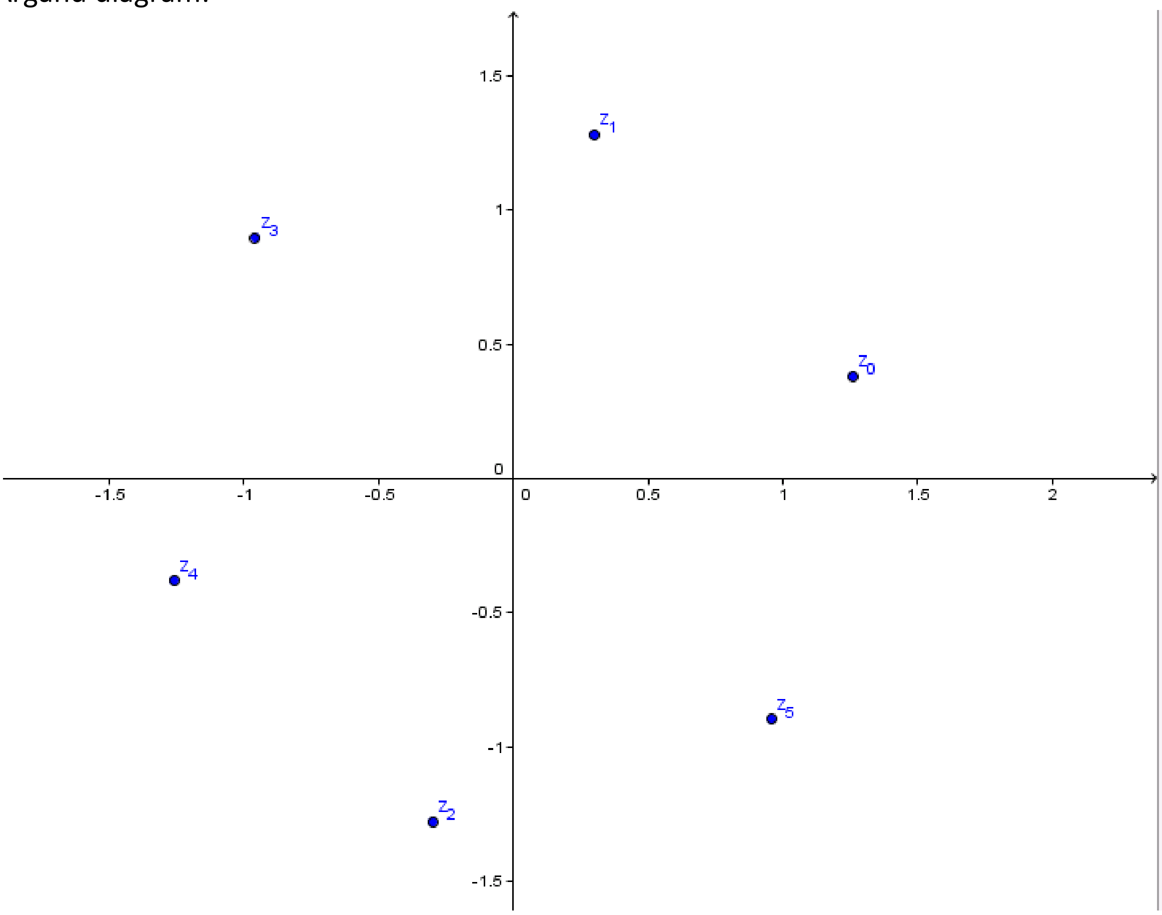
Use De Moivre's theorem to obtain the 6th roots of i5 – 1. Also show them in an Argand diagram

Solution.

If $z = -1 + 5i$, then $|z| = \sqrt{1^2 + 5^2} = \sqrt{26}$.
Therefore $z = \sqrt{26}\left(-\frac{1}{\sqrt{26}} + \frac{5}{\sqrt{26}}i\right)$. We need to find $\varphi \in (-\pi; \pi]$ such that $\cos\varphi = -\frac{1}{\sqrt{26}}$ and $\sin\varphi = \frac{5}{\sqrt{26}}$. Therefore $\frac{\pi}{2} < \varphi < \pi$ and $\tan\varphi = \frac{\sin\varphi}{\cos\varphi} = -5$. Hence $\varphi = -\arctan 5 + \pi$, as $-\frac{\pi}{2} < -arctan\,5 < 0$. By De Moivre's theorem the 6th roots are

$$z_i = \sqrt[12]{26}\left(\cos\frac{-\arctan 5 + \pi + 2\pi k}{6} + i\sin\frac{-\arctan 5 + \pi + 2\pi k}{6}\right),$$

where $k = 0,1,2,3,4,5$..
The Argand diagram:



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where $k = 0,1,2,3,4,5$.