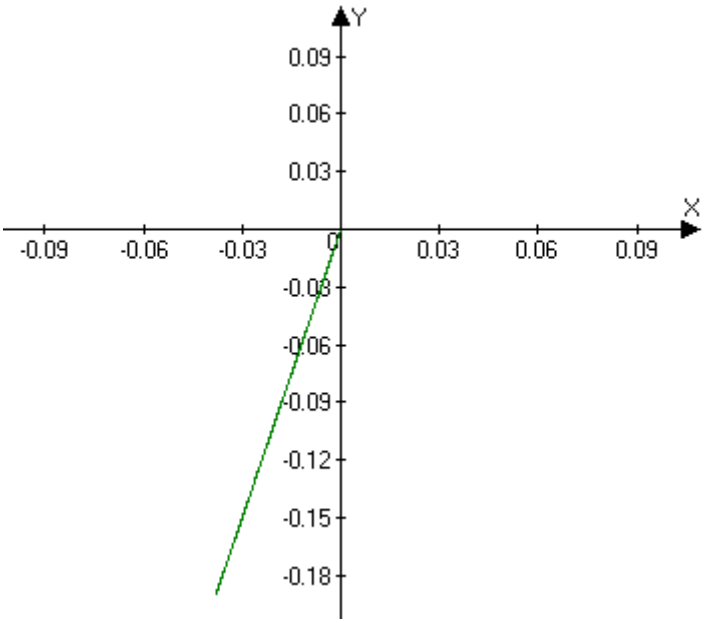
Answer on Question #44536 – Math - Complex Analysis

Obtain the geometric, polar and exponential representations of $\frac{1}{(i5-1)}$.

Solution

$$z=\frac{1}{(5i-1)}=\frac{(5i+1)}{(5i-1)(5i+1)}=\frac{(5i+1)}{-25-1}=\frac{(5i+1)}{-26}=-\frac{1}{26}-\frac{5}{26}i$$

Geometric representation $= x + yi$:



Polar representation:

$z = r(\cos\varphi + i\sin\varphi),\quad r = |z|$

$$r=\sqrt{x^2+y^2}=\sqrt{\left(-\frac{1}{26}\right)^2+\left(-\frac{5}{26}\right)^2}=\sqrt{\frac{1}{676}+\frac{25}{676}}=\sqrt{\frac{26}{676}}=\sqrt{\frac{1}{26}}$$

$$z=\sqrt{\frac{1}{26}}\left(-\sqrt{\frac{1}{26}}+i\left(-\sqrt{\frac{25}{26}}\right)\right)$$

$$\varphi=\operatorname{atan}\left(\frac{-\frac{5}{26}}{-\frac{1}{26}}\right)-\pi=\operatorname{atan}(5)-\pi$$

$$z=\sqrt{\frac{1}{26}}(\cos(\operatorname{atan}(5)-\pi)+i\sin(\operatorname{atan}(5)-\pi))$$

Exponential representation:

$$z=re^{i\varphi}$$

$$z=\sqrt{\frac{1}{26}}e^{i(\operatorname{atan}(5)-\pi)}$$