

Answer on Question #44529 – Math - Abstract Algebra

Problem.

Factorise 10 in two ways in $\mathbb{Z}[p]$

Solution.

We suppose that $p > 10$, as there are no elements with residue 10 in $\mathbb{Z}[p]$, when $p \leq 10$.

$10 = 2 \cdot 5$, $10 = -2 \cdot -5$ in \mathbb{Z} . To find two different factorization of 10 in $\mathbb{Z}[p]$ we should replace -5 , -2 , 2 , 5 and 10 with their residue by modulo p . We will obtain that $10 \equiv 2 \cdot 5 \pmod{p}$ and $10 \equiv (p - 2) \cdot (p - 5) \pmod{p}$.

$2 \not\equiv p - 2 \pmod{p}$, $5 \not\equiv p - 5 \pmod{p}$, $2 \not\equiv p - 5 \pmod{p}$, $5 \not\equiv p - 2 \pmod{p}$, as $p > 10$.

Answer: $10 \equiv 2 \cdot 5 \pmod{p}$ and $10 \equiv (p - 2) \cdot (p - 5) \pmod{p}$.