Answer on Question #44527 – Math – Abstract Algebra:

Find two different Sylow 2-subgroups of D_{12}.

Solution.

$$|D_{12}| = 2 \cdot 12 = 24 = 2^3 \cdot 3;$$

So, if M, N are Sylow 2-subgroups, then $|M| = |N| = 2^3 = 8$.

Note that D_n has the following representation:

$$D_n =< x, y | x^n = y^2 = e, xy = yx^{-1} >;$$

So:

$$D_{12} = \{e, x, x^2, \dots, x^{11}, y, xy, x^2y, \dots x^{11}y\};$$

$$\forall\ i = 0, \dots, 11: x^i y = x^{i-1} \cdot yx^{-1} = \cdots = yx^{-i};$$

$$\forall\ i, j = 0, \dots, 11: x^i y \cdot x^j y = x^i y \cdot yx^{-j} = x^{i-j};$$

Consider the following subgroups:

$$M = \{e, x^3, x^6, x^9, y, x^3y, x^6y, x^9y\};$$

$$N = \{e, x^3, x^6, x^9, xy, x^4y, x^7y, x^{10}y\};$$

$|M| = |N| = 8$, so M and N are Sylow 2-subgroups of D_{12}. Note that the group $C_4 = \{e, x^3, x^6, x^9\}$ is a subgroup of index 2 of M and N. Hence, $M \cong N \cong C_4 \times C_2$.