Answer on Question #44524 – Math - Other

Problem.

Which of the following statements are true? Give reasons for your answers. (This means that if you think a statement is false, give a short proof or an example that shows it is false. If it is true, give a short proof for saying so. For instance, to show that '{1, padma, blue}is a set' is true, you need to say that this is true because it is a well-defined collection of 3 objects.)

i) {MTE-04, –3, Indira Gandhi} is a set.

ii) For any two sets A and B, A ∪ Bc = A ∩ B.

iii) There is a unique z ∈ C for which z z 1 − = .

iv) The least degree of the polynomial with real coefficients and with roots 2+i, 2i – 1 is 2.

v) If a statement has a direct proof, then it cannot be proved by contradiction.

vi) The equation x = 3 has the same geometric representation regardless of whether it is an equation in one variable or two variables.

vii) Any system of n linear equations in n – 1 variables has a solution.

viii) The CS inequality is a generalization of the triangle inequality.

Remark.

The statement isn't correctly formatted. I suppose that the correct statement is

"Which of the following statements are true? Give reasons for your answers. (This means that if you think a statement is false, give a short proof or an example that shows it is false. If it is true, give a short proof for saying so. For instance, to show that '{1, padma, blue}is a set' is true, you need to say that this is true because it is a well-defined collection of 3 objects.)

i) {MTE-04, –3, Indira Gandhi} is a set.

ii) For any two sets A and B, $A \cup B^c = A \cap B. A \cup B^c$

iii) There is a unique $z \in \mathbb{C}$ for which $|\bar{z}| = |z^{-1}|$.

iv) The least degree of the polynomial with real coefficients and with roots $2 + i$, $2i - 1$ is 2.

v) If a statement has a direct proof, then it cannot be proved by contradiction.

vi) The equation $x = 3$ has the same geometric representation regardless of whether it is an equation in one variable or two variables.

vii) Any system of n linear equations in $n - 1$ variables has a solution.

viii) The CS inequality is a generalization of the triangle inequality. "

Solution.

i) True

{MTE-04, –3, Indira Gandhi} is a set, as it is a well-defined collection of 3 objects.

ii) False

Suppose that $A = [0; 1]$ and $B = [-2; -1]$ are subsets of universe $U = \mathbb{R}$. Then $B^c = (-\infty; -2) \cup (-1; +\infty)$, $A \cup B^c = (-\infty; -2) \cup (-1; +\infty)$, but $A \cap B = \emptyset$.

iii) False

There are at least two such numbers, as $|\bar{1}| = |1^{-1}|$ and $|\overline{-1}| = |(-1)^{-1}|$.

iv) False

If $a + ib$ is the root of polynomial with real coefficients$p(x)$, then $a - ib$ is the root of polynomial $p(x)$. Hence polynomial with roots $2 + i$, $2i - 1$ has also root $2 - i$ and $-1 - 2i$. Therefore it has degree at least 4.

v) True

If suppose that statement is incorrect, then from direct proof we will obtain a contradiction.

vi) False

If $x = 3$ is an equation in one variable, then its geometric representation is point. If $x = 3$ is an equation in two variables, then its geometric representation is line.

vii) False

The system $\begin{cases} x + y = 1 \\ x + y = 2 \\ x - y = 0 \end{cases}$ doesn't have solution. $x + y$ couldn't be equal to 1 and 2 at one time.

viii) False

CS inequality and triangle inequalities are equivalent in Hilbert spaces (like $\mathbb{R}^n$ with standard metric), but the inner product isn't defined in all metric spaces.