Answer on Question #44523 – Math - Other

Problem.

Which of the following statements are true? Give reasons for your answers. (This means that if you think a statement is false, give a short proof or an example that shows it is false. If it is true, give a short proof for saying so. For instance, to show that '{1, padma, blue} is a set' is true, you need to say that this is true because it is a well-defined collection of 3 objects.)

i) For any two sub sets A and B of a set U, A×B=B×A.

ii) The contrapositive of 'If Lalita is good at mathematics, then her daughter passes the maths exam' is the statement 'If Lalita is not good at mathematics, then her daughter fails the maths exam'.

iii) The geometric representations of z and z−1
(for z ≠ 0, z∈C) are the reflections of each other
in the x-axis.

iv) Arg .
4 4
π
=
π

v) Any polynomial over Q can have at most one root in C \ R.

vi) If a∈R, then det ([a]) ≥ .0

vii) If 1 10 α ,…,α are positive real numbers with arithmetic mean A, then their harmonic mean is A−1
.

viii) Any two linear equations in the variables x and y will have infinitely many common solutions.

ix) {Descartes, DeMorgan, ϕ}is a set.

x) A matrix representation of x+y−z=3 is [x y z] −1 11 = [3].

Remark.

The statement isn't correctly formatted. I suppose that the correct statement is

"Which of the following statements are true? Give reasons for your answers. (This means that if you think a statement is false, give a short proof or an example that shows it is false. If it is true, give a short proof for saying so. For instance, to show that '{1, padma, blue} is a set' is true, you need to say that this is true because it is a well-defined collection of 3 objects.

i) For any two sub sets A and B of a set U, $A \times B = B \times A$.

ii) The contrapositive of 'If Lalita is good at mathematics, then her daughter passes the maths exam' is the statement 'If Lalita is not good at mathematics, then her daughter fails the maths exam'.

iii) The geometric representations of z and z^{-1} (for $z \neq 0$, $z \in \mathbb{C}$) are the reflections of each other in the x-axis.

iv) $\mathrm{Arg}\frac{\pi}{4} = \frac{\pi}{4}$.

v) Any polynomial over $\mathbb{Q}$ can have at most one root in $\mathbb{C}\backslash\mathbb{R}$.

vi) If $a \in \mathbb{R}$, then $\det([a]) \geq 0$.

vii) If $\alpha_1, \ldots, \alpha_{10}$ are positive real numbers with arithmetic mean A, then their harmonic mean is A^{-1}.

viii) Any two linear equations in the variables x and y will have infinitely many common solutions.

ix) {Descartes, DeMorgan, ϕ} is a set.

x) A matrix representation of $x + y - z = 3$ is $[x \quad y \quad z]\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = [3]$."

Solution.

i) False
If $A = \{1\}$ and $B = \{2\}$, then $A \times B = \{(1,2)\}$ and $B \times A = \{(2,1)\}$, but $\{(1,2)\} \neq \{(2,1)\}$.

ii) False
The contrapositive to 'If *p*, then *q.*' is 'If not *q*, then not *p*.'.

iii) True
If $z = a + ib$, then $z^{-1} = a - ib$. The geometric representation of z and z^{-1} are points (a, b) and $(a, -b)$. They are the reflections of each other in the x-axis.

iv) False
$\mathrm{Arg}\frac{\pi}{4} = \mathrm{Arg}\left(\frac{\pi}{4}(\cos 0 + i \sin 0)\right) = 0 \neq \frac{\pi}{4}$.

v) False
The polynomial $x^2 + 1 = (x - i)(x + i)$ has two roots in $\mathbb{C}\backslash\mathbb{R}$.

vi) True
The rows of matrix $[a]$ are linearly dependent (as they are equal). Hence $\det([a]) = 0$. Therefore $\det([a]) \geq 0$.

vii) False
If $a_i = 1$ for $i = 1..5$ and $a_i = 4$ for $i = 6..10$, then $\mathrm{AM} = \frac{a_1 + \cdots + a_{10}}{10} = \frac{5+5\cdot 4}{10} = 2.5$ and $\mathrm{GM} = \sqrt[10]{a_1 \ldots a_{10}} = \sqrt[10]{4^5} = 2$, but $(2.5)^{-1} \neq 2$.

viii) False
The system of two equation $\begin{cases} x - y = 0; \\ x + y = 2, \end{cases}$ has one solution $x = y = 1$.

ix) True
{Descartes, DeMorgan, ϕ} is a set, as it is a well-defined collection of 3 objects.

x) True
If $a_1 x_1 + \cdots a_n x_n = b$ is linear equation, then $[x_1 \quad \ldots \quad x_n]\begin{bmatrix} a_1 \\ \ldots \\ a_n \end{bmatrix} = [b]$ is its linear representation.