Answer on Question #44504 – Math - Trigonometry

Problem.

THS IS FUNCTIONS (CHAPTER ;RELATION BETWEEN SIDES AND ANGle or using sine formula show that each of the following holdand the law of sine rule

Q1 .tan A-B/2/TAN A+B/2=a-b/a+b
Q2 .b cos B+c cos C=a cos (B-C)
Q3 a sin B-C/2\=(b-c) cos A/2
Q4 b+c/b-c=tan B+C/2 * COT B-C/2
Q5 a cosA+b cos C+c cos C=2a sinB sinC
Q5. IN ANY TRIANGLE IF a/ cosA=b/cosB, PROVE THAT THE TRIANGLE IS ISOSCELES

Remark.

The statement isn't correctly formatted. I suppose that the correct statement is

"Q1. $\frac{\tan\frac{A-B}{2}}{\tan\frac{A+B}{2}} = \frac{a-b}{a+b}$

Q2. $b\cos B\ + c\cos C\ = a\cos\ (B-C)$

Q3 $a\sin\frac{B-C}{2}\ = (b-c)\cos\frac{A}{2}$

Q4 $\frac{b+c}{b-c}c = \tan\frac{B+C}{2}\cdot\cot\frac{B-C}{2}$

Q5 $a\cos A + b\cos B + c\cos C = 2\,a\sin B\sin C$ (I suppose that should be $b\cos B$ instead of $b\cos C$)

Q6. In any triangle if $\frac{a}{cosA} = \frac{b}{cosB}$, then the triangle is isosceles"

Solution.

Suppose that a, b, c are the sides of triangle, A, B, C are the degree measures of the angles and R is the radius of circumcircle. By the Law of sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R.$$

Therefore $a = 2R\sin A$, $b = 2R\sin B$, $c = 2R\sin C$.

Q1

$$\frac{a-b}{a+b} = \frac{2R\sin A - 2R\sin B}{2R\sin A + 2R\sin B} = \frac{\sin A - \sin B}{\sin A + \sin B} = \frac{2\sin\frac{A-B}{2}\cos\frac{A+B}{2}}{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}} = \frac{\tan\frac{A-B}{2}}{\tan\frac{A+B}{2}}.$$

Q2

$$b\cos B\ + c\cos C\ = 2R\sin B\cos B + 2R\sin C\cos C\ = R(\sin 2B + \sin 2C)$$
$$= 2R\sin(B+C)\cos(B-C)$$
$$= 2R\sin(\pi - A)\cos(B-C) = 2R\sin A\cos(B-C) = a\cos(B-C)$$

Q3

$$a\sin\frac{B-C}{2} = 2R\sin A\sin\frac{B-C}{2} = 2R\sin(\pi - B - C)\sin\frac{B-C}{2} = 4R\sin(B+C)\sin\frac{B-C}{2}$$
$$= 4R\sin\frac{B+C}{2}\cos\frac{B+C}{2}\sin\frac{B-C}{2} = 2R\ \cos\frac{\pi - B - C}{2}(\sin B - \sin C)$$
$$= \cos\frac{A}{2}(2R\sin B - 2R\sin C) = \cos\frac{A}{2}(b-c).$$

Q4 From Q1 if $b \neq c$

$$\frac{b+c}{b-c}=\frac{\tan\frac{B+C}{2}}{\tan\frac{B-C}{2}}=\tan\frac{B+C}{2}\cdot\cot\frac{B-C}{2}.$$

Q5 From Q2

$$a\cos A+b\cos B+c\cos C=a\cos A+a\cos(B-C)=a(\cos(\pi-B-C)+\cos(B-C))$$
$$=a(\cos(B-C)-\cos(B+C))=-2a\sin B\sin -C=2a\sin B\sin C.$$

Q6 $\frac{a}{\cos A}=\frac{b}{\cos B}$ if and only if $\frac{2R\sin A}{\cos A}=\frac{2R\sin B}{\cos B}$ or $\tan A=\tan B$. $\tan A=\tan B$ if and only if $A=B$, as $\tan x$ has period π.