

Answer on Question #44456 – Math - Abstract Algebra

Problem.

Let $s = 1\ 2\ 3\ 4\ 5\ 6\ 7$

$2\ 4\ 5\ 6\ 7\ 3\ 1$ and $t = 1\ 2\ 3\ 4\ 5\ 6\ 7$

$3\ 2\ 4\ 1\ 6\ 5\ 7$ be elements of S_7 .

- Write both s and t as product of disjoint cycles and as a product of transpositions,
- Find the signatures of s and t .
- Compute ts^{-2} and t^2s^2 .

Remark.

The statement isn't correctly formatted. I suppose that the correct statement is

"Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 6 & 7 & 3 & 1 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 1 & 6 & 5 & 7 \end{pmatrix}$ be elements of S_7 :

- Write both σ and τ as product of disjoint cycles and as a product of transpositions.
- Find the signatures of σ and τ .
- Compute $\tau\sigma^{-2}$ and $\tau^2\sigma^2$."

Solution.

$$\text{i) } \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 6 & 7 & 3 & 1 \end{pmatrix} = (1\ 2\ 4\ 6\ 3\ 5\ 7) = (1\ 2)(2\ 4)(4\ 6)(6\ 3)(3\ 5)(5\ 7);$$

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 1 & 6 & 5 & 7 \end{pmatrix} = (1\ 3\ 4)(5\ 6) = (1\ 3)(3\ 4)(5\ 6);$$

$$\text{ii) } \text{sgn}(\sigma) = (-1)^6 = 1 \text{ and } \text{sgn}(\tau\tau) = (-1)^3 = 1.$$

$$\text{iii) } \sigma^{-2} = (\sigma^{-1})^2 = ((1\ 7\ 5\ 3\ 6\ 4\ 2))^2 = (1\ 5\ 6\ 2\ 7\ 3\ 4).$$

$$\text{Therefore } \tau\sigma^{-2} = (1\ 3)(3\ 4)(5\ 6)(1\ 5\ 6\ 2\ 7\ 3\ 4) = (1\ 6\ 2\ 7\ 4\ 3).$$

$$\sigma^2 = (1\ 2\ 4\ 6\ 3\ 5\ 7)^2 = (1\ 4\ 3\ 7\ 2\ 6\ 5) \text{ and } \tau = ((1\ 3)(3\ 4)(5\ 6))^2 = e.$$

$$\text{Therefore } \tau^2\sigma^2 = (1\ 4\ 3\ 7\ 2\ 6\ 5).$$