Answer on Question #44432 – Math – Trigonometry

$tan(\theta)=\frac{sin(\alpha)-cos(\alpha)}{\sin(\alpha)+cos(\alpha)}$ Then $sin(\alpha)+cos(\alpha)=?$

Solution

$$tan(\theta)=\frac{tan(\alpha)-1}{\tan(\alpha)+1}=1-\frac{2}{tan(\alpha)+1},$$

Rewrite

$$\frac{2}{tan(\alpha)+1}=1-tan(\theta),$$

$$tan(\alpha)+1=\frac{2}{1-tan(\theta)}$$

$$tan(\alpha)=\frac{2}{1-tan(\theta)}-1$$

$$tan(\alpha)=\frac{2-1+tan(\theta)}{1-tan(\theta)}$$

$$tan(\alpha)=\frac{1+tan(\theta)}{1-tan(\theta)}$$

$$tan^2(\alpha)=\left(\frac{1+tan(\theta)}{1-tan(\theta)}\right)^2$$

$$tan^2(\alpha)+1=\left(\frac{1+tan(\theta)}{1-tan(\theta)}\right)^2+1=\frac{\left(1+tan(\theta)\right)^2+\left(1-tan(\theta)\right)^2}{\left(1-tan(\theta)\right)^2}=\frac{2\left(1+tan^2(\theta)\right)}{\left(1-tan(\theta)\right)^2} \qquad (1)$$

It is known that $tan^2(\alpha)+1=\frac{1}{cos^2(\alpha)}$, hence

$cos^2(\alpha)=\frac{1}{tan^2(\alpha)+1}$, substitute expression (1) and obtain

$cos^2(\alpha)=\frac{\left(1-tan(\theta)\right)^2}{2\left(1+tan^2(\theta)\right)}$,

It is known

$$sin^2(\alpha)=1-cos^2(\alpha)=1-\frac{\left(1-tan(\theta)\right)^2}{2\left(1+tan^2(\theta)\right)}=\frac{2\left(1+tan^2(\theta)\right)-\left(1-tan(\theta)\right)^2}{2\left(1+tan^2(\theta)\right)}$$

$$sin^2(\alpha)=\frac{1+tan^2(\theta)+2tan(\theta)}{2\left(1+tan^2(\theta)\right)}=\frac{\left(1+tan(\theta)\right)^2}{2\left(1+tan^2(\theta)\right)}$$

If $sin(\alpha)>0, cos(\alpha)>0$, then

$$\sin(\alpha) + \cos(\alpha) = \sqrt{\frac{(1+\tan(\theta))^2}{2(1+\tan^2(\theta))}} + \sqrt{\frac{(1-\tan(\theta))^2}{2(1+\tan^2(\theta))}} = \frac{|1+\tan(\theta)|+|1-\tan(\theta)|}{\sqrt{2(1+\tan^2(\theta))}}$$

If $\sin(\alpha) > 0, \cos(\alpha) < 0$, then

$$\sin(\alpha) + \cos(\alpha) = \sqrt{\frac{(1+\tan(\theta))^2}{2(1+\tan^2(\theta))}} - \sqrt{\frac{(1-\tan(\theta))^2}{2(1+\tan^2(\theta))}} = \frac{|1+\tan(\theta)|-|1-\tan(\theta)|}{\sqrt{2(1+\tan^2(\theta))}}$$

If $\sin(\alpha) < 0, \cos(\alpha) > 0$, then

$$\sin(\alpha) + \cos(\alpha) = -\sqrt{\frac{(1+\tan(\theta))^2}{2(1+\tan^2(\theta))}} + \sqrt{\frac{(1-\tan(\theta))^2}{2(1+\tan^2(\theta))}} = \frac{-|1+\tan(\theta)|+|1-\tan(\theta)|}{\sqrt{2(1+\tan^2(\theta))}}$$

If $\sin(\alpha) < 0, \cos(\alpha) < 0$, then

$$\sin(\alpha) + \cos(\alpha) = -\sqrt{\frac{(1+\tan(\theta))^2}{2(1+\tan^2(\theta))}} - \sqrt{\frac{(1-\tan(\theta))^2}{2(1+\tan^2(\theta))}} = \frac{-|1+\tan(\theta)|-|1-\tan(\theta)|}{\sqrt{2(1+\tan^2(\theta))}}$$