Answer on Question #44402 – Math - Analytic Geometry

Find n so that vectors 2i+3j-2k,5i+nj +k and -i+2j+3k may be coplanar

Solution:

$a = < 2, 3, -2 >$
$b = < 5, n, 1 >$
$c = < -1, 2, 3 >$

Vectors are coplanar if and only if the cross product of two is perpendicular to the third, i.e. if and only if

$$(a \times b) \cdot c = 0.$$

This is the scalar triple product, which is basically equivalent to taking a determinant. Performing this computation, we get:

$$a \times b = < a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x >=$$
$$= < 3 \cdot 1 + 2 \cdot n, -2 \cdot 5 - 2 \cdot 1, 2 \cdot n - 3 \cdot 5 > = < 3 + 2n, -12, -15 + 2n >$$

$$(a \times b) \cdot c = < (a \times b)_x c_x + (a \times b)_y c_y + (a \times b)_z c_z >=$$
$$= < 3 + 2n, -12, -15 + 2n > \cdot < -1, 2, 3 >=$$
$$= -2n + 3(2n - 15) - 27 = 4n - 72 = 0$$
$$n = \frac{72}{4} = 18$$

Answer: vectors are coplanar when $n = 18$.