Answer on Question #44356 – Math - Abstract Algebra

Problem.

5) a) Let H = h(1 2)i and K = h(1 2 3)i be subroups of S3. Check that S3 = HK. Is S3 the internal direct product of H and K? Justify your answer.
b) Let s = 1 2 3 4 5 6 7
2 4 5 6 7 3 1and t = 1 2 3 4 5 6 7
3 2 4 1 6 5 7be elements of S7.
i) Write both s and t as product of disjoint cycles and as a product of transpositions,
ii) Find the signatures of s and t.
iii) Compute ts

Remark.

The statement isn't correctly formatted. I suppose that the correct statement is
"5) a) Let $H = \langle(1\ 2)\rangle$ and $K = \langle(1\ 2\ 3)\rangle$ be subroups of S_3. Check that $S_3 = HK$. Is S_3 the internal direct product of H and K? Justify your answer.
b) Let $s = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 6 & 7 & 3 & 1 \end{pmatrix}$ and $t = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 1 & 6 & 5 & 7 \end{pmatrix}$ be elements of S_7.
i) Write both s and t as product of disjoint cycles and as a product of transpositions,
ii) Find the signatures of s and t.
iii) Compute ts."

Solution.

a) The elements of H are $\{e, (1\ 2)\}$.
The elements of K are $\{e, (1\ 2\ 3), (1\ 3\ 2)\}$
There 6 elements in S_3 each of it could be presented as product elements from H and K.
$e = ee$;
$(1\ 2) = (1\ 2)e$;
$(1\ 3) = (1\ 2)(1\ 3\ 2)$;
$(2\ 3) = (1\ 2)(1\ 2\ 3)$;
$(1\ 2\ 3) = e(1\ 2\ 3)$;
$(1\ 3\ 2) = e(1\ 3\ 2)$.

b)
i) $s = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 6 & 7 & 3 & 1 \end{pmatrix} = (1\ 2\ 4\ 6\ 3\ 5\ 7) = (1\ 2)(2\ 4)(4\ 6)(6\ 3)(3\ 5)(5\ 7)$.
$t = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 1 & 6 & 5 & 7 \end{pmatrix} = (1\ 3\ 4)(5\ 6) = (1\ 3)(3\ 4)(5\ 6)$.
ii) $\operatorname{sgn}(s) = (-1)^6 = 1$ and $\operatorname{sgn}(t) = (-1)^3 = 1$.
iii) $ts = (1\ 3\ 4)(5\ 6)(1\ 2\ 4\ 6\ 3\ 5\ 7) = (1\ 2)(3\ 6\ 4\ 5\ 7)$.