Answer on Question #44352 – Math – Abstract Algebra

Question:

Consider the set of matrices

$$G = \left\{ a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};\ b = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\ c = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix};\ d = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix};\ e = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}; f = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}; \right\}$$

with coefficients in Z2.

a) Make the Cayley table and check that this set forms a group with respect to matrix multiplication. (You can assume that matrix multiplication is associative.)

b) Find the orders of all the elements in G.

c) Show that the group is isomorphic to S3 by giving an isomorphism f : G -> S3.

Solution.

a) The Cayley table for G is the following:

G/*	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	a	f	e	d	c
c	c	e	a	f	b	d
d	d	f	e	a	c	b
e	e	c	d	b	f	a
f	f	d	b	c	a	e

Let us see if set G forms a group with respect to matrix multiplication:

- G is associative.
- neutral element or 1 is matrix a
- and every element of G has inverse element, which can be easily find from the Cayley table.

b) b*b=a, hence the order of b is 2;

c*c=a , hence the order of c is 2;

d*d=a, hence the order of d is 2;

e*e=f; e*e*e=f*e=a, hence the order of e is 3;

f*f=e; f*f*f=e*f=a, hence the order of f is 3.

c) The Cayley table for S3 is the following:

Element	**123**	**213**	**132**	**321**	**231**	**312**
123	123	213	132	321	231	312
213	213	123	231	312	132	321
132	132	312	123	231	321	213
321	321	231	312	123	213	132
231	231	321	213	132	312	123
312	312	132	321	213	123	231

And comparing this two table we see that, the isomorphism between this groups is:

a -> 123

b -> 132

c -> 213

d -> 321

e ->231

f -> 312