Answer on Question #44342 – Math - Statistics and Probability

Problem.

A statistical process analyst is responsible for assuring statistical control. In one process, a machine is supposed to drop 11.4 ounces of mints into a bag. (Assume that this process can be approximated by a normal distribution). The acceptable ranges for weights of the bags of mints are 11.25 ounces to 11.55 ounces, inclusive.

An error with the release valve has caused the setting on the mint release machine to “shift”. Assume that the machine shift is filling the bags with a mean weight of 11.56 ounces and a standard deviation of 0.05 ounces. To check that the machine is placing the correct weight of mints into the bags, you randomly select three samples of five bags each and find the mean weight in ounces for each sample.

Respond to the following:

1.While sampling individual bags:

◦You randomly select a bag of mints. What is the probability that the bag you select is not outside the acceptable range? (That is, you do not detect that the machine has shifted.)

◦You randomly select 15 bags of mints. What is the probability that you select at least one bag that is not outside the acceptable range?

2.While sampling groups of five:

◦You randomly select a sample of five bags. What is the probability that your sample of five bags has a mean that is not outside the acceptable range? (That is, you do not detect that the machine has shifted.)

◦You randomly select three samples of five bags. What is the probability that you select at least one sample of five bags that has a mean that is not outside the acceptable range?

3.Describe your solutions to each of these problems and explain whether taking 15 bags of mints in one sample or three samples of five bags each is the better way to test for a process that is out of statistical control.

Solution.

1. The filling of bags is distributed normally with mean $\mu = 11.56$ ounces and a standard deviation of $\sigma = 0.05$ ounces. The corresponding transformation formula is $Z = \frac{X-\mu}{\sigma}$. Z is distributed normally, $Z \sim N(0,1)$. The probability the bag we select is not outside the acceptable range equals

$$p_1 = P(11.25 < X < 11.55) = P(-6.2 < Z < -0.2) \approx 0.42.$$

The probability that all 15 bags we select are outside the acceptable range equals

$$p_2 = (1 - p_1)^{15} \approx 0.0002.$$

Therefore the probability that at least one bag that is not outside the acceptable range

$$p_3 = 1 - p_2 \approx 0.9998.$$

2. The filling of bags is distributed normally with mean $\mu = 11.56$ ounces and a standard deviation of $\sigma = 0.05$ ounces. The sample of $n = 5$ bags is distributed normally $X_3 \sim N(\mu, \frac{\sigma}{\sqrt{n}})$. The corresponding transformation formula is $Z = \frac{X_3-\mu}{\sigma/\sqrt{n}}$. Z is distributed normally, $Z \sim N(0,1)$. The probability the sample we select has a mean that is not outside the acceptable range equals

$$p_4 = P(11.25 < X_3 < 11.55) = P(-13.86 < Z < -0.45) \approx 0.33.$$

The probability that all three samples we select has a mean that is outside the acceptable range equals

$$p_5 = (1 - p_4)^3 \approx 0.036.$$

Therefore the probability that at least one sample has a mean that is not outside the acceptable range equals

$$p_6 \approx 0.964.$$

3. Taking 15 bags of mints in one sample is better, as probability that we find the bag that is outside the acceptable range is higher $p_3 > p_6$.