Answer on Question #44340 – Math - Linear Algebra

**Problem**

Standard basis vectors for R^3 are (1,0,0),(0,1,0) and (0,0,1). If we want to insert u → into this basic, then which vector from standard basis can be removed while still maintaining the basis of R^3.
Discuss the case when:
u → =(4,3,6)
u → =(4,0,6)
Interpret the result geometrically in both cases.

**Solution**

1) $\vec{u} = (4, 3, 6)$.

$\vec{u} = 4(1,0,0) + 3(0,1,0) + 6(0,0,1)$, thus we can remove each vector from standard basis and each of $(\vec{u}, \overrightarrow{e_1}, \overrightarrow{e_2})$, $(\vec{u}, \overrightarrow{e_1}, \overrightarrow{e_3})$, $(\vec{u}, \overrightarrow{e_3}, \overrightarrow{e_2})$ will form the basis of $\mathbb{R}^3$.
Geometrical interpretation: none of $Oxy, Oyz, Oxz$ contains $\vec{u}$.

2) $\vec{u} = (4, 0, 6)$.

$\vec{u} = 4(1,0,0) + 6(0,0,1)$, which means that $(\vec{u}, \overrightarrow{e_1}, \overrightarrow{e_3})$ are not linearly independent, thus we can remove only $\overrightarrow{e_1}$ or $\overrightarrow{e_3}$ not $\overrightarrow{e_2}$. So, $(\vec{u}, \overrightarrow{e_1}, \overrightarrow{e_2})$, $(\vec{u}, \overrightarrow{e_3}, \overrightarrow{e_2})$ can form the basis of $\mathbb{R}^3$.
Geometrical interpretation: neither $Oxy$ nor $Oyz$ contains $\vec{u}$, but $\vec{u}$ lies in $Oxz$.