Answer on Question #44337 – Math - Differential Calculus | Equations

Solve dy/dx = (y+x-2) /(y-x-4)

Solution:

We make the change of variables:

$$\begin{cases} x = \xi + x_0 \\ y = \eta + y_0 \end{cases}$$

where x_0 and y_0 are the solutions of linear system of equations

$$\begin{cases} y_0 + x_0 - 2 = 0 \\ y_0 - x_0 - 4 = 0 \end{cases}$$

Rewrite the system in matrix form and solve it by Gaussian Elimination

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 4 \end{bmatrix}$$

from 2 rows we subtract the 1-th row, multiplied respectively by 1

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 2 \end{bmatrix}$$

divide the 2-th row by -2

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

from 1 rows we subtract the 2-th row

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix}$$

Answer:

$$\begin{cases} x_0 = -1 \\ y_0 = 3 \end{cases}$$

For new variables we have $dx = d\xi, dy = d\eta$ and $d\eta/d\xi = (\eta + \xi)\ /(\eta - \xi)$.
We make the change of variable z= η/ξ, then $\eta = \xi * z$ and $d\eta/d\xi = z + \xi * dz/d\xi$.
Now for variable z we obtain differential equation:
$z + \xi * dz/d\xi = (z + 1)\ /(z - 1)$
collecting terms containing z we obtained equation with separable variables
$\xi * dz/d\xi = (-z^2 + 2z + 1)\ /(z - 1)$.
Separating variables

$$\frac{z-1}{-z^2+2z+1} * dz = \frac{d\xi}{\xi}.$$

Integrating the left-hand side of the equality with respect to z and right side with respect to ξ:

$\int \frac{z-1}{-z^2+2z+1}\, dz = \int \frac{-2z+2}{-2(-z^2+2z+1)}\, dz = \int \frac{d(-z^2+2z+1)}{-2(-z^2+2z+1)} = -\frac{1}{2}*\ln|-z^2+2z+1|$+C,

$\int \frac{d\xi}{\xi} = \ln|\xi|$+C

We have

$-\frac{1}{2}*\ln|-z^2+2z+1| = \ln|\xi| + \ln C.$

(we denote arbitrary constant C by lnC)

Using the properties of logarithms ln(x):

$$\ln\left|\frac{1}{\sqrt{-z^2+2z+1}}\right| = \ln|\xi * C|$$

from here obtained

$$\frac{1}{\sqrt{-z^2+2z+1}} = \xi * C$$

or rewrite equality by $C_1=(-z^2+2z+1)*\xi^2$, where C_1 arbitrary constant.

Back to old variables

$$z=\frac{\eta}{\xi}=\frac{y-y_0}{x-x_0}=\frac{y-3}{x+1}$$

We have

$C_1 = (x+1)^2 * \left(-\left(\frac{y-3}{x+1}\right)^2 + 2\left(\frac{y-3}{x+1}\right) + 1\right) = -(y-3)^2 + 2(y-3)(x+1) +$
$+(x+1)^2 = -y^2 + 2xy + x^2 + 8y - 4x - 14.$

denote arbitrary constant $C_1 + 14$ by C, then

Answer:

$-y^2 + 2xy + x^2 + 8y - 4x = C$, where C arbitrary constant, is a solution of differential equation dy/dx = (y+x-2) /(y-x-4).