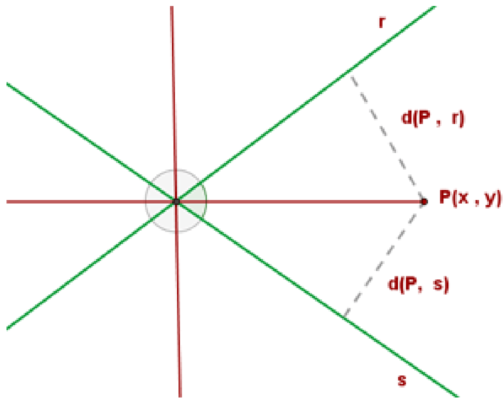
Answer on Question #44317 – Math – Other

Find the equation of the angle bisectors of the 3x+4y=5 and 12x+5y=5. Also find the bisector which bisect the acute angle.

Solution:

The bisector of an angle is the locus of points on the plane that are equidistant from the rays that form the angle.

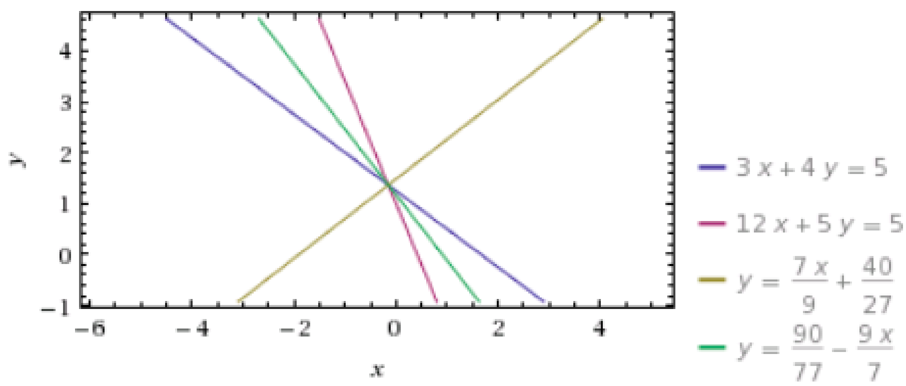


$$d(P, r) = d(P, s)$$
$$r: A_1x + B_1y + C_1 = 0$$
$$r: 3x + 4y - 5 = 0$$
$$s: A_2x + B_2y + C_2 = 0$$
$$s: 12x + 5y - 5 = 0$$
$$P(x, y)$$
$$\frac{|A_1x + B_1y + C_1|}{\sqrt{A_1^2 + B_1^2}} = \frac{|A_2x + B_2y + C_2|}{\sqrt{A_2^2 + B_2^2}}$$

Two angle bisectors are perpendicular between themselves.

$$\frac{|3x + 4y - 5|}{\sqrt{3^2 + 4^2}} = \frac{|12x + 5y - 5|}{\sqrt{12^2 + 5^2}}$$
$$5(12x + 5y - 5) = 13(3x + 4y - 5)$$
$$21x + 40 = 27y$$
$$y = \frac{7}{9}x + \frac{40}{27}$$

$$5(12x + 5y - 5) = -13(3x + 4y - 5)$$
$$99x + 77y = 90$$
$$y = -\frac{9}{7}x + \frac{90}{77}$$



Now let's take one of the given lines and let its slope be m_1 and take one of the bisectors and let its slope be m_2.

If θ be the acute angle between them, then find $tan\ \theta\ =\ \left|\frac{m_1-m_2}{1+m_1 m_2}\right|$
If $\tan\theta\ >\ 1$ then the bisector taken is the bisector of the obtuse angle and the other one will be the bisector of the acute angle.

First line $3x+4y-5=0$ has a slope of $-\frac{3}{4}$
Then $m_1\ =-\frac{3}{4}$

$y=-\frac{9}{7}x+\frac{90}{77}$ is bisector, that has a slope of $-\frac{9}{7}$
Then $m_2\ =-\frac{9}{7}$

$$tan\ \theta\ =\ \left|\frac{m_1-m_2}{1+m_1 m_2}\right|=\left|\frac{-\frac{3}{4}-\left(-\frac{9}{7}\right)}{1+\left(-\frac{3}{4}\right)\left(-\frac{9}{7}\right)}\right|=\frac{3}{11}$$

$$\frac{3}{11}<1$$

Thus, $y=-\frac{9}{7}x+\frac{90}{77}$ is bisector which bisect this acute angle.