Answer on Question #44283 – Math - Linear Algebra

Problem.

which sets are a basis for the following vector subspace of P2 :
X={A e M22 : A [1] = [0] }
[2] [0]

A {[2 0] , [0 -1] , [0 0] , [0 0] } C {[2 -1] , [0 0] }
[0 0] [0 0] [2 0] [0 -1] [0 0] [2 -1]

B{[2 -1] } D { [2 -1] , [2 -1] }
[2 -1] [2 -1] [-2 1]

Remark. I suppose that the correct statement is

"Which sets are a basis for the following vector subspace of P_2: $X = \left\{A \in M_{22}: A\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}\right\}$

A $\left\{\begin{bmatrix}2 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & -1\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\2 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\0 & -1\end{bmatrix}\right\}$

B $\left\{\begin{bmatrix}2 & -1\\2 & -1\end{bmatrix}\right\}$

C $\left\{\begin{bmatrix}2 & -1\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\2 & -1\end{bmatrix}\right\}$

D $\left\{\begin{bmatrix}2 & -1\\2 & -1\end{bmatrix}, \begin{bmatrix}2 & -1\\-2 & 1\end{bmatrix}\right\}$

Solution.

If $A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$, then $\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$ or $a + 2b = 0$ and $c + 2d = 0$, hence $a = -2b,\ c = -2d$.

Therefore

$$A = \begin{bmatrix}a & b\\c & d\end{bmatrix} = \begin{bmatrix}-2b & b\\-2d & d\end{bmatrix} = -b\begin{bmatrix}2 & -1\\0 & 0\end{bmatrix} - d\begin{bmatrix}0 & 0\\2 & -1\end{bmatrix}.$$

So the basis for the given vector subspace is $\left\{\begin{bmatrix}2 & -1\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\2 & -1\end{bmatrix}\right\}$.

The correct answer is C.

Answer: C.