Answer on Question #44256, Math, Trigonometry

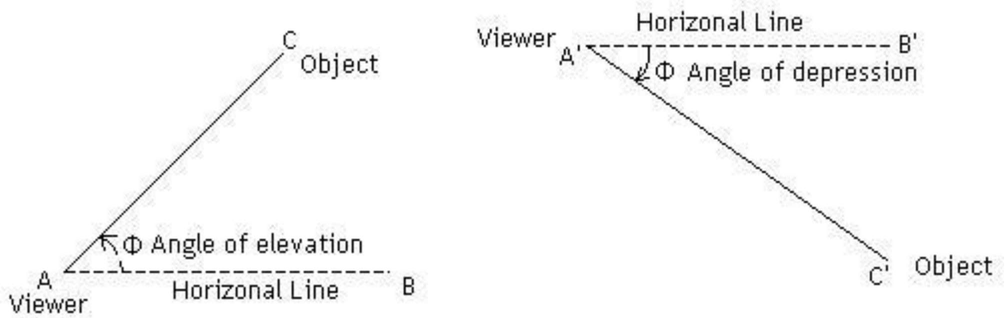
The angle of elevation of a tower from a point L is 62°. From a point K, 50 m further from the tower, the angle of elevation is 47°. (Let the height of the tower be h.)

a Use the sine rule in ΔKTL to show that: TL =
b Use trigonometry in ΔLMT to show that: TL =
c Hence, show that h =
d Calculate the height, h, of the tower, correct to one decimal place.

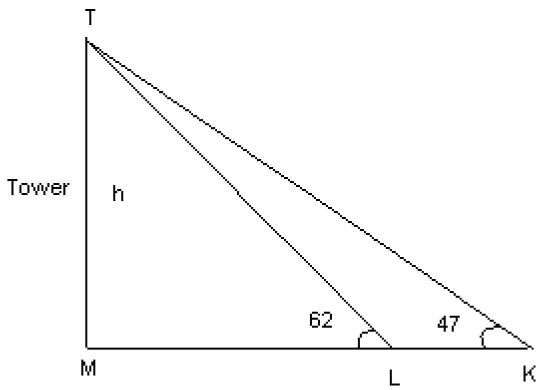
15 From the top of a cliff, the angles of depression of two boats at sea 0.5 km apart are 55° and 33°. (Let the height of the cliff be h.)
a Show that the height of the cliff is: h =
b Hence, calculate the height, correct to the nearest metre.

Solution:

Angles of elevation and depression (Φ) are formed by the horizontal lines that a viewer's lines of sight form to an object.



For the first problem we have drawing:



TM=h (height of the tower), MK=50 m (distance from a point K to tower)

a) In trigonometry, **sine rule** is an equation relating the lengths of the sides to the sines of its angles. According to the law

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

where *a*, *b*, and *c* are the lengths of the sides of a triangle, and *A*, *B*, and *C* are the opposite angles.

Hence in ΔKTL from sine rule: $\frac{\sin \angle KTL}{LK}=\frac{\sin \angle TKL}{TL}$

$\angle KTL$=180°-$\angle TKL$-(180°-$\angle TLM$) =180°-47°-(180°-62°)=15°. From here we obtain

TL=LK*$\frac{\sin 47°}{\sin 15°}$

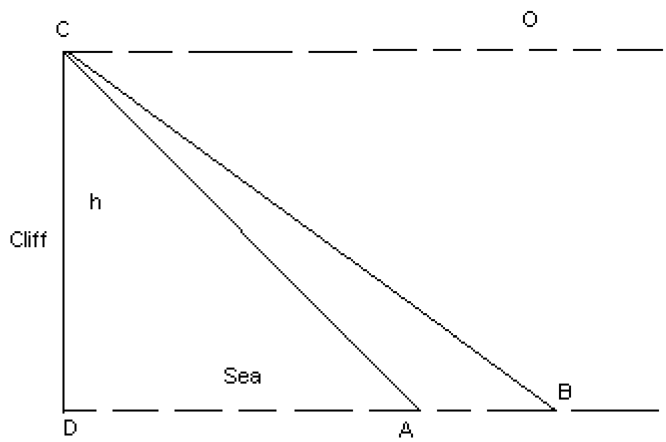
b) in ΔLMT $\angle TML$=90°, hence TL =$\frac{TM}{\sin \angle TLM}$=$\frac{TM}{\sin 62°}$=$\frac{h}{\sin 62°}$

c) from b) we obtain h=TL*$\sin 62°$, also from ΔKMT $\frac{TM}{MK}$=tan$47°$,

hence h=MK* tan$47°$.

d) h=MK* tan$47°$=50*1.072=53.6 (m)

Second problem:



A-first boat, B-second boat, AB=0.5 km distance between boats

$\angle OCB$=33°, $\angle OCA$=55° the angles of depression of two boats.

Lines CO and DA are parallel, hence $\angle BAC$=180°-$\angle OCA$=180°-55°=125°.

In ΔABC from sine rule: $\frac{\sin \angle BAC}{BC}=\frac{\sin \angle ACB}{AB}$, and BC=$\frac{\sin \angle BAC}{\sin \angle ACB}$*AB.

$\angle ACB = \angle OCA - \angle OCB =$22°. In ΔCDB $\angle CDB =$90°, $\angle DBC =$90°-33°=57°.

Hence

a)h=CD=BC* cos$\angle DBC$=$\frac{\sin \angle BAC}{\sin \angle ACB}$*AB* cos$\angle DBC$

b) from a) obtain h=0.5*$\frac{\sin 125°}{\sin 22°}$* cos57°=0.595(km)