Answer on Question #44245, Math, Complex Analysis

w=(3-i)/(2i-1) solving this i get that w= -i-1 now i want to express w in polar form: r=sqrt2 but can you explain why arg(w)= -3pi/4 ? when arctan(1) = pi/4 (how do they get -3pi/4) and if arg(w)= -3pi/4 can i than write 5pi/4 instead and z^4=w and how to draw the solutions in the complex plane.

Solution.

$$w = \frac{3-i}{2i-1} = \frac{(3-i)(-2i-1)}{(2i-1)(-2i-1)} = \frac{-6i-3-2+i}{1+4} = \frac{-5-5i}{5} = -1-i.$$

The modulus equals $|w| = \sqrt{1^2+1^2} = \sqrt{2}$. Hence $w = \sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$. The argument of z is the value of ϕ for which $\cos\phi = -\frac{1}{\sqrt{2}}$ and $\sin\phi = -\frac{1}{\sqrt{2}}$. The $\frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = \tan\frac{\pi}{4} = \tan\left(-\frac{3\pi}{4}\right) = \frac{\sin\left(-\frac{3\pi}{4}\right)}{\cos\left(-\frac{3\pi}{4}\right)} = 1$, but $\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \neq -\frac{1}{\sqrt{2}}$. The set of values ϕ is $\left\{-\frac{3\pi}{4} + 2\pi n : n \in \mathbb{Z}\right\}$. To make $\arg w$ a well-defined function, it is defined as function which equals the value of ϕ in the open-closed (closed-open) interval of the length 2π. Authors often use $(-\pi;\pi]$, but in some literature you may find interval $(0;2\pi]$, the value in this interval is often denoted as $\operatorname{Arg}(w)$. Hence you may write that $w = \sqrt{2}\left(\cos\frac{5\pi}{4} + \sin\frac{5\pi}{4}\right)$, but $\arg(w) \neq \frac{5\pi}{4}$.

From the n-th root formula the solution of the equation $z^4 = w$.

$$z_0 = \sqrt[8]{2}\left(\sin\left(-\frac{3\pi}{16}\right) + i\cos\left(-\frac{3\pi}{16}\right)\right);$$

$$z_1 = \sqrt[8]{2}\left(\sin\left(\frac{\frac{-3\pi}{4}+2\pi}{4}\right) + i\cos\left(\frac{\frac{-3\pi}{4}+2\pi}{4}\right)\right) = \sqrt[8]{2}\left(\sin\left(\frac{5\pi}{16}\right) + i\cos\left(\frac{5\pi}{16}\right)\right);$$

$$z_2 = \sqrt[8]{2}\left(\sin\left(\frac{\frac{-3\pi}{4}+4\pi}{4}\right) + i\cos\left(\frac{\frac{-3\pi}{4}+4\pi}{4}\right)\right) = \sqrt[8]{2}\left(\sin\left(\frac{13\pi}{16}\right) + i\cos\left(\frac{13\pi}{16}\right)\right);$$

$$z_3 = \sqrt[8]{2}\left(\sin\left(\frac{\frac{-3\pi}{4}+6\pi}{4}\right) + i\cos\left(\frac{\frac{-3\pi}{4}+6\pi}{4}\right)\right) = \sqrt[8]{2}\left(\sin\left(\frac{21\pi}{16}\right) + i\cos\left(\frac{21\pi}{16}\right)\right)$$

$$= \sqrt[8]{2}\left(\sin\left(-\frac{11\pi}{16}\right) + i\cos\left(-\frac{11\pi}{16}\right)\right).$$

The roots of the equation $z^4 = w$ lie in the vertices of the squares with a diagonal $2|z| = 2\sqrt[8]{2}$ and center at $(0;0)$. The square is rotated at angle $\frac{5\pi}{16}$ on the center of the square.