Answer on Question #44244 – Math - Statistics and Probability

Suppose that a random sample of nine measurements from a normally distributed population gives a sample mean of _ = 2.57 and a sample standard deviation of s = .3. Use critical values to test H0 : μ = 3 versus Ha : μ ≠3 using levels of significance α = .10, α = .05, α = .01, and α = .001.

Solution:

$$H_o : \mu = 3.00, H_a : \mu \neq 3.00$$

Two-tailed test.

Since $n < 30$, population is normal, unknown population variance, use t-test.

Critical region is of the form $\{|T| > t\}\ or\ \{T < -t, T > t\}, T\ is\ a\ t - distribution, df = n - 1 = 8.$

Value of test statistic,

$$t = \frac{2.57 - 3.00}{\frac{0.3}{\sqrt{(9)}}} = -\frac{0.43}{0.1} = -4.3.$$

We have a T-table that gives areas for $P\{T > t\}$, for row $df = 8$ value 4.3 is between

$3.36\ and\ 4.50$, with probabilities $0.001\ and\ 0.005$ respectively.

Now we can write:

$$0.001 < P\{T > 4.3\} < 0.005, to\ give\ 0.002 < P\{|T| > 4.3\} < 0.01.$$

The sample value $t = -4.3$ can occur with a probability between $0.002\ and\ 0.01$.

We have the following results:

$P\{T > 4.3\} < 0.01$; significant at α = 0.01 (also at 0.10 and 0.05)

$P\{T > 4.3\} > 0.002 > 0.001$; not significant at α = 0.001

Conclusion: we reject H_o. (α = 0.1, α = 0.01, α =0.05).

At α = 0.001 we don’t reject H_o.