Answer on Question #44235 – Math - Statistics and Probability

In order to control costs, a company wishes to study the amount of money its sales force spends entertaining clients. The following is a random sample of six entertainment expenses (dinner costs for four people) from expense reports submitted by members of the sales force.

$157 $132 $109 $145 $125 $139

a Calculate _ , s2 and s for the expense data. In addition, show that the two different formulas for calculating s2 give the same result.

b. Assuming that the distribution of entertainment expenses is approximately normally distributed, calculate estimates of tolerance intervals containing 68.26 percent, 95.44 percent, and 99.73 percent of all entertainment expenses by the sales force.

c. If a member of the sales force submits an entertainment expense (dinner cost for four) of $190, should this expense be considered unusually high (and possibly worthy of investigation by the company)? Explain your answer.

d. Compute and interpret the z-score for each of the six entertainment expenses.

Solution

a.

$$\bar{x} = \frac{157 + 132 + 109 + 145 + 125 + 139}{6} = \$134.5$$

$$s^2 = \frac{(157 - 134.5)^2 + (132 - 134.5)^2 + (109 - 134.5)^2 + (145 - 134.5)^2 + (125 - 134.5)^2 + (139 - 134.5)^2}{5}$$
$$= 276.7$$

$$s = \sqrt{(s^2)} = \sqrt{(276.7)} = \$\,16.634$$

Two different formulas for calculating s^2:

$$s^2 = \frac{\sum X_i^2 - \frac{(\sum X_i)^2}{n}}{n-1}.$$

$$\sum X_i^2 = (157)^2 + (132)^2 + (109)^2 + (145)^2 + (125)^2 + (139)^2 = 109925$$

$$\sum X_i = 157 + 132 + 109 + 145 + 125 + 139 = 807$$

$$s^2 = \frac{109925 - \frac{(807)^2}{6}}{5} = 276.7$$

$$s^2 = \frac{\sum X_i^2}{n-1} - \frac{n}{n-1}\bar{x}^2 = \frac{109925}{5} - \frac{6}{5}(134.5)^2 = 276.7$$

b.

69.26% of the expense would fall within 1 standard deviation from the mean :

$$134.5 \pm 16.634 = (\$117.866, \$\,151.134)$$

95.45% of the expense would fall within 2 standard deviations from the mean :

$$134.5 \pm 2(16.634) = (\$101.232, \$167.768)$$

99.73% of the expense would fall within 3 standard deviations from the mean :

$$134.5 \pm 3(16.634) = (\$84.598, \$184.402)$$

c.

Yes. An expense of $190 is considered as unusually high because it is outside of the interval of 3 standard deviations from the mean.

d.

z-score for 157 is

$$z = \frac{157 - 134.5}{16.634} = 1.35$$

An expense of $157 is 1.35 standard deviations above the average expense.

z-score for 132 is

$$z = \frac{132 - 134.5}{16.634} = -0.15$$

An expense of $132 is 0.15 standard deviations below the average expense.

z-score for 109 is

$$z = \frac{109 - 134.5}{16.634} = -1.53$$

An expense of $109 is 1.53 standard deviations below the average expense.

z-score for 145 is

$$z = \frac{145 - 134.5}{16.634} = 0.631$$

An expense of $145 is 0.631 standard deviations above the average expense.

z-score for 125 is

$$z = \frac{125 - 134.5}{16.634} = -0.57$$

An expense of $125 is 0.57 standard deviations below the average expense.

z-score for 139 is

$$z = \frac{139 - 134.5}{16.634} = 0.27$$

An expense of $139 is 0.27 standard deviations above the average expense.