Answer on Question 44229 - Math -Complex Analysis

It's necessary to solve

$$\mathrm{Im}(-z+i) = (z+i)^2$$

Due to the definition, $z \in \mathbb{C}, z = x + iy; x, y \in \mathbb{R}$; $\mathrm{Im}\, z = \mathrm{Im}(x+iy) = y$. Having substituted $z = x + iy$ and the definition to equation, we have:

$$\mathrm{Im}(-z+i) = \mathrm{Im}(-x-iy+i) = \mathrm{Im}(-x+i(1-y)) = 1-y.$$

$$(z+i)^2 = z^2+2iz+i^2 = z^2+2iz-1 = (x+iy)^2+2i(x+iy)-1 = x^2+2ixy-y^2+2ix-2y-1$$

$$(z+i)^2 = x^2-(y^2+2y+1)+i(2xy+2x) = x^2-(y+1)^2+i(2xy+2x)$$

Left and right sides of the equation coincide, and both of sides are complex numbers, so we have the following:

$$\begin{cases} 1-y = x^2-(y+1)^2 \\ 2xy+2x = 0 \end{cases}$$

This system must be solved. From the second equation,

$$2x(1+y) = 0.$$

$$\left[\begin{array}{l} x = 0 \\ y = -1. \end{array}\right.$$

If $x = 0$, then from the first equation:

$$1-y = -(y+1)^2$$

$$1-y = -y^2-2y-1$$

$$y^2+y+2 = 0$$

This equation has no real roots. If $y = -1$, then from the first equation

$$2 = x^2$$

$$x = \pm\sqrt{2}.$$

So, the answer is $(\pm\sqrt{2}; -1) = \pm\sqrt{2} - i$