Answer on Question #44204 – Math - Real Analysis

Find the supremum and infimum of the set {(-1)^n(1+1/n): n is natural

Solution

Under even *n* the terms of this sequence become $\left(1+\frac{1}{n}\right)$, *n=2, 4, 6,…*

Under odd *n* the terms of this sequence become $-\left(1+\frac{1}{n}\right)$, *n=1, 3, 5,…*

We note that every element of the set is less than 2 since

$(-1)^n\left(1+\frac{1}{n}\right) \le \left(1+\frac{1}{n}\right) < 2$.

We claim that the supremum (the least upper bound) is 2.

Assume that 2 is not the least upper bound. Then there is an $\varepsilon > 0$ such that $2-\varepsilon$ is also an upper bound. However, we claim that there is a natural number n such that

$2-\varepsilon < 1+\frac{1}{n}$,

This inequality is equivalent with the following sequence of inequalities

$1-\varepsilon < \frac{1}{n}$, $\frac{1}{n} > 1-\varepsilon$.

If $\varepsilon < 1$ then $0 < n < \frac{1}{1-\varepsilon}$.

If $\varepsilon \ge 1$ then $n > 0$.

There is a natural number n such that

$2-\varepsilon < 1+\frac{1}{n}$,

So, $2-\varepsilon$ is not an upper bound for the set. This verifies our answer.

We note that every element of the set is greater than or equal to -2:

$$(-1)^n\left(1+\frac{1}{n}\right) \ge -\left(1+\frac{1}{n}\right) \ge -2$$

If a set has a minimum, then the minimum will also be an infimum. Infimum of the set is -2.

Answer: 2 and -2.