Answer on Question #44202 – Math - Statistics and Probability

Problem

The mean weight of chicken in a chicken dinner at a fast food restraint is 10 ounces with standard deviation of 0.5 ounces. Using the distribution of sample means, what is the probability that the average chicken weight in a sample of 100 dinners will differ from the mean by more than 0.03 ounces?

Solution

The weight of chicken is a random variable with mean $\mu = 10$ and standard deviation $\sigma = 0.5$.
By central limit theorem, whatever the population, the distribution of $\bar{X}_n = \frac{X_1+X_2+\cdots+X_n}{n}$ is approximately normal when $n = 100$ is large.
It is known the average chicken weight in a sample of n =100 dinners is a random variable with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$.

$\bar{X}_n = \frac{X_1+X_2+\cdots+X_n}{n} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$, i.e. $\bar{X}_{100} \sim N(10,0.05)$.

Consequently, the next variable is a standard normal with mean 0 and standard deviation 1: $Z = \frac{\bar{X}_{100}-E(\bar{X}_{100})}{\sqrt{D(\bar{X}_{100})}} = \frac{\bar{X}_{100}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$.

Calculate

$$P(\bar{X}_{100} - 10 > 0.03) = P(\bar{X}_{100} > 10.03) = 1 - P(\bar{X}_{100} \le 10.03) =$$
$$= 1 - P\left(\frac{\bar{X}_{100} - E(\bar{X}_{100})}{\sqrt{D(\bar{X}_{100})}} \le \frac{10.03 - E(\bar{X}_{100})}{\sqrt{D(\bar{X}_{100})}}\right) =$$
$$= 1 - P\left(\frac{\bar{X}_{100} - E(\bar{X}_{100})}{\sqrt{D(\bar{X}_{100})}} \le \frac{10.03 - 10}{\frac{0.5}{10}}\right) \approx 1 - P\left(Z \le \frac{0.03}{0.05}\right) =$$
$$= 1 - P(Z \le 0.6) = 1 - 0.7257 = 0.2743 ;$$
$$P(\bar{X}_{100} - 10 < -0.03) = P(\bar{X}_{100} < 9.97) = P\left(\frac{\bar{X}_{100} - E(\bar{X}_{100})}{\sqrt{D(\bar{X}_{100})}} < \frac{9.97 - E(\bar{X}_{100})}{\sqrt{D(\bar{X}_{100})}}\right) =$$
$$= P\left(\frac{\bar{X}_{100} - E(\bar{X}_{100})}{\sqrt{D(\bar{X}_{100})}} < \frac{9.97 - 10}{\frac{0.5}{10}}\right) \approx P\left(Z \le \frac{-0.03}{0.05}\right) = P(Z \le -0.6) =$$
$$= 0.2743$$

By additive property of probability, taking into account events $(\bar{X}_{100} - 10 > 0.03)$ and $(\bar{X}_{100} - 10 < -0.03)$ are mutually exclusive, therefore,

$$P(|\bar{X}_{100} - 10| > 0.03) = P\big((\bar{X}_{100} - 10 > 0.03) \cup (\bar{X}_{100} - 10 < -0.03)\big) =$$
$$= P(\bar{X}_{100} - 10 > 0.03) + P(\bar{X}_{100} - 10 < -0.03) = 0.2743 + 0.2743 = 0.5486 \approx 0.549.$$

Answer. 0.549.