

Answer on Question #44181 - Math - Statistics and Probability

Case Study 1. Suppose that Body Mass Index (BMI) for a population of 30-60-year-old men follows a Normal distribution with mean 26, and standard deviation 4. Please calculate the range of BMI that 95% of subjects fall within? For a randomly selected 30-60-year old man, what is the probability that he is obese (i.e. BMI ≥ 30)?

Solution:

a). To calculate the range of BMI that 95% of subjects fall within we need to determine left and right probability borders centered at mean (50 %). The borders will be 2.5% and 97.5% and the probability of 95% will be the area under the curve colored in blue (see Fig.1).

Using standard table of normal distribution, we calculate z-scores for given probabilities:

For $p = 0.025$ z-score is $z = -1.96$

For $p = 0.975$ z-score is $z = 1.96$

Using calculated z-score, we determine left and right borders using the formula $z = (x - \text{mean}) / \text{sd}$.

$x_1 = z_1 * \text{sd} + \text{mean} = -1.96 * 4 + 26 = 18.16$

$x_2 = z_2 * \text{sd} + \text{mean} = 1.96 * 4 + 26 = 33.84$

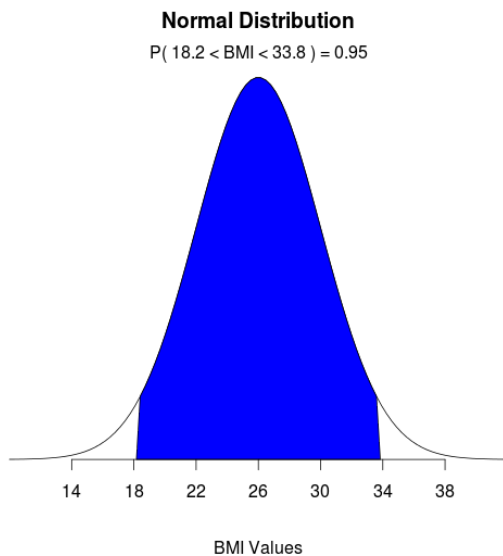


Fig 1.

b) To determine probability of BMI ≥ 30 we need to calculate z-score for this value:

$$z = (x - \text{mean}) / \text{sd} = (30 - 26) / 4 = 1$$

Then, using standard table of normal distribution, we find the probability: 0.8413

This is the probability that randomly selected 30-60-year old man has BMI < 30 . The probability that he has BMI ≥ 30 is:

$$1 - 0.8413 = 0.1587$$

Answer:

a) the range is from 18.16 to 33.84

b) the probability is 0.1587 or 15.87%

R code:

a) `qnorm(0.025, 26, 4); qnorm(0.975, 26, 4);`

b) `1 - pnorm(30, 26, 4)`