

Answer on Question #44177, Math, Trigonometry

if $K = \sin(\pi/18) \cdot \sin(5\pi/18) \cdot \sin(7\pi/18)$
then find value of k

Solution.

$$\begin{aligned} K &= \sin\left(\frac{\pi}{18}\right) \sin\left(\frac{5\pi}{18}\right) \sin\left(\frac{7\pi}{18}\right) \\ &= \sin\left(\frac{\pi}{2} - \frac{8\pi}{18}\right) \sin\left(\frac{\pi}{2} - \frac{4\pi}{18}\right) \sin\left(\frac{\pi}{2} - \frac{2\pi}{18}\right) = \cos\left(\frac{8\pi}{18}\right) \cos\left(\frac{4\pi}{18}\right) \cos\left(\frac{2\pi}{18}\right) \\ &= \cos\left(\frac{2\pi}{18}\right) \cos\left(\frac{4\pi}{18}\right) \cos\left(\frac{8\pi}{18}\right) = \frac{2 \sin\left(\frac{2\pi}{18}\right) \cos\left(\frac{2\pi}{18}\right) \cos\left(\frac{4\pi}{18}\right) \cos\left(\frac{8\pi}{18}\right)}{2 \sin\left(\frac{2\pi}{18}\right)} \\ &= \frac{\sin\left(\frac{4\pi}{18}\right) \cos\left(\frac{4\pi}{18}\right) \cos\left(\frac{8\pi}{18}\right)}{2 \sin\left(\frac{2\pi}{18}\right)} = \frac{2 \sin\left(\frac{4\pi}{18}\right) \cos\left(\frac{4\pi}{18}\right) \cos\left(\frac{8\pi}{18}\right)}{4 \sin\left(\frac{2\pi}{18}\right)} \\ &= \frac{\sin\left(\frac{8\pi}{18}\right) \cos\left(\frac{8\pi}{18}\right)}{4 \sin\left(\frac{2\pi}{18}\right)} = \frac{2 \sin\left(\frac{8\pi}{18}\right) \cos\left(\frac{8\pi}{18}\right)}{8 \sin\left(\frac{2\pi}{18}\right)} = \frac{\sin\left(\frac{16\pi}{18}\right)}{8 \sin\left(\frac{2\pi}{18}\right)} = \frac{\sin\left(\pi - \frac{2\pi}{18}\right)}{8 \sin\left(\frac{2\pi}{18}\right)} \\ &= \frac{\sin\left(\frac{2\pi}{18}\right)}{8 \sin\left(\frac{2\pi}{18}\right)} = \frac{1}{8}. \end{aligned}$$

Answer. $K = \frac{1}{8}$.