Answer on Question #44153 – Math - Statistics and Probability

The head of XYZ Rent-a-Car believes that the mean number of miles between services is 2150 miles with standard deviation 440 miles. If he is correct, what is the probability that the mean of a sample of 36 cars would be less than 2324 miles?

Solution

Because the population is normal, the distribution of the sample mean $\bar{X} = \frac{X_1+X_2+X_3+\cdots+X_{36}}{36}$ is exactly normal with mean 2150 miles and standard deviation $sd(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{440}{\sqrt{36}} = \frac{440}{6} = \frac{220}{3}$ miles.

Let $Z = \frac{\bar{X}-2150}{\frac{220}{3}}$, Z is the standard normal variable with zero (miles) mean and standard deviation equals 1 (miles).

Because $\bar{X}$ is $N\left(2150, \frac{220}{3}\right)$, therefore

$$P(\bar{X} < 2324) = P\left(\frac{\bar{X}-2150}{\frac{220}{3}} < \frac{2324-2150}{\frac{220}{3}}\right) = P(Z < 2.372)=0.9911.$$

The last probability is calculated by means of statistical tables of standard normal probabilities or software.

About ninety nine cars in one hundred ones would the mean number of miles be less than 2324 miles.

Answer: 0.9911.