

Answer on Question #44097 – Math – Integral Calculus

Integral of $\sin^3(2x)/(1+\cos(2x))$

Solution

$$\begin{aligned}
 \int \frac{\sin^3(2x)dx}{1+\cos(2x)} &= \int \frac{\sin^2(2x) \cdot \sin(2x)dx}{1+\cos(2x)} = -\frac{1}{2} \int \frac{\sin^2(2x) \cdot (-\sin(2x))d(2x)}{1+\cos(2x)} = \\
 &|use d(2x) = 2d(x), \quad d(\cos(2x)) = -2 \sin(2x) dx| = \\
 &= -\frac{1}{2} \int \frac{\sin^2(2x) d(\cos(2x))}{1 + \cos(2x)} = \\
 &= -\frac{1}{2} \int \frac{(1 - \cos^2(2x))d(\cos(2x))}{1 + \cos(2x)} = |use a^2 - b^2 = (a - b)(a + b)| \\
 &= -\frac{1}{2} \int \frac{(1 - \cos(2x))(1 + \cos(2x))d(\cos(2x))}{1 + \cos(2x)} = |use \int -f(x)dx = - \int f(x)dx| \\
 &= \frac{1}{2} \int (-(1 - \cos(2x)))d(\cos(2x)) = \\
 &= |use d(-\cos(2x)) = -d(\cos(2x))| = \frac{1}{2} \int (1 - \cos(2x)) d(-\cos(2x)) = \\
 &|use d(f(x) + g(x)) = d(f(x)) + d(g(x)) = f'(x)dx + g'(x)dx, (1)' = 0| = \\
 &= \frac{1}{2} \int (1 - \cos(2x)) d(1 - \cos(2x)) = |substitution t = 1 - \cos(2x)| = \frac{1}{2} \int t dt = \frac{1}{2} \frac{t^2}{2} + C = \\
 &\frac{(1-\cos(2x))^2}{4} + C = \frac{(2\sin^2(x))^2}{4} + C = \frac{4\sin^4(x)}{4} + C = \sin^4(x) + C,
 \end{aligned}$$

where C is an arbitrary real constant. Moreover, $\cos(2x) \neq -1, 2x \neq \pi + 2k\pi, k$ is integer,

$x \neq \frac{\pi}{2} + k\pi, k$ is integer.

Answer: $\sin^4(x) + C$