Answer on Question #44096, Math, Complex Analysis

Problem 1.

$$\sqrt{-2i} = \sqrt{2}e^{-i\frac{\pi}{4}} = 1 - i$$

Solution 1.

$$\sqrt{-2i} = \sqrt{2\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)} = \sqrt{2}\left(\cos\frac{-\frac{\pi}{2} + 2\pi k}{2} + i\sin\frac{-\frac{\pi}{2} + 2\pi k}{2}\right), k = 0,1.$$

When $k = 0$: $\sqrt{-2i} = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right) = \sqrt{2}e^{-i\frac{\pi}{4}}$.

On the other hand, $1 - i = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right) = \sqrt{2}e^{-i\frac{\pi}{4}}$.

Problem 2.

w^2= (-1+sqrt3 i)/2

Solution 2.

$w^2 = \frac{-1+\sqrt{3}i}{2}$;

assume that $w = a + bi$, then $w^2 = a^2 + 2abi - b^2$;

$a^2 + 2abi - b^2 = \frac{-1+\sqrt{3}i}{2}$;

$$\begin{cases} a^2 - b^2 = -\frac{1}{2}, \\ 2ab = \frac{\sqrt{3}}{2}; \end{cases}$$

$$\begin{cases} a^2 - b^2 = -\frac{1}{2}, \\ a = \frac{\sqrt{3}}{4b}; \end{cases}$$

$$\begin{cases} \frac{3}{16b^2} - b^2 = -\frac{1}{2}, \\ a = \frac{\sqrt{3}}{4b}; \end{cases}$$

$$\begin{cases} 3 - 16b^4 + 8b^2 = 0, \\ a = \frac{\sqrt{3}}{4b}; \end{cases}$$, substitute $t = b^2$;

$$\begin{cases} 16t^2 - 8t - 3 = 0, \\ a = \frac{\sqrt{3}}{4b}, \\ t = b^2; \end{cases}$$

$$\begin{cases} t_{1,2} = \frac{4 \pm \sqrt{16 + 16 \cdot 3}}{16} = \frac{4 \pm 8}{16} , \\ a = \frac{\sqrt{3}}{4b}, \\ t = b^2; \end{cases}$$

$$\begin{cases} b_1^2 = \frac{3}{4}, \\ b_2^2 = -\frac{1}{4}, \\ a = \frac{\sqrt{3}}{4b}; \end{cases}$$

$$\left[\begin{array}{l}\begin{cases}b=\frac{\sqrt{3}}{2},\\ a=\frac{1}{2}\end{cases}\\ \begin{cases}b=-\frac{\sqrt{3}}{2},\\ a=-\frac{1}{2}\end{cases}\end{array}\right.$$

$w_{1,2}=\pm\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$.

P.S. and yes, $\tan^{-1}\left(-\sqrt{3}\right)=\frac{2\pi}{3}$, as you mentioned, because $\tan\varphi=\frac{y}{x}$, and $y=\frac{\sqrt{3}}{2}, x=-\frac{1}{2}$.

You can solve it not in algebraic form, though: $\frac{-1+\sqrt{3}i}{2}=\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)$ and $w^2=r^2(\cos 2\alpha+i\sin 2\alpha)$. But algebraic form seems to be easier way to solve this problem.