Answer on Question #44093 – Math – Integral Calculus

Integral of sin^3 x/(1+cos x)

Solution

$$\int \frac{sin^3\, x\, dx}{1+\cos(x)} = \int \frac{sin^2 x \cdot \sin(x) dx}{1+\cos(x)} =- \int \frac{sin^2 x \cdot (-\sin(x)) dx}{1+\cos(x)} = |use\ \ d(\cos(x)) = -\sin(x)\, dx| =$$

$$= -\int \frac{sin^2 x\, d(\cos(x))}{1+\cos(x)} =$$

$$= -\int \frac{(1-cos^2 x) d(\cos(x))}{1+\cos(x)} = |use\ a^2 - b^2 = (a-b)(a+b)|$$

$$= -\int \frac{(1-\cos(x))(1+\cos(x)) d(\cos(x))}{1+\cos(x)} = \left|use\ \int -f(x)dx = -\int f(x)dx\right|$$

$$= \int \big(-(1-\cos(x))\big) d(\cos(x)) =$$

$$= |use\ d(-\cos(x)) = -d(\cos(x))| = \int (1-\cos(x))\, d(-\cos(x)) = \big|use\ d\big(f(x)+g(x)\big) = d\big(f(x)\big) + d\big(g(x)\big) = f'(x)dx + g'(x)dx, (1)' = 0\big| == \int (1-\cos(x))\, d(1-\cos(x)) =$$

$$|substitution\ \ t = 1-\cos(x)| = \int t dt = \frac{t^2}{2} + C = \frac{(1-\cos(x))^2}{2} + C = \frac{\left(2sin^2\left(\frac{x}{2}\right)\right)^2}{2} + C = \frac{4sin^4\left(\frac{x}{2}\right)}{2} + C =$$

$$= 2sin^4\left(\frac{x}{2}\right) + C,$$

where C is an arbitrary real constant. Moreover, $\cos(x) \neq -1$, $x \neq \pi + 2k\pi$, k is integer.

Answer: $2sin^4\left(\frac{x}{2}\right) + C$