Answer on Question #44084 – Math - Vector Calculus

a ⃗,b ⃗,c ⃗are non-zero vectors. If a ⃗×b ⃗= a ⃗×c ⃗ and a ⃗ .b ⃗= a ⃗ .c ⃗ then show that b ⃗= c ⃗.

Solution

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b}).$$
$$\vec{a} \times (\vec{b} - \vec{c}) = \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \vec{0}.$$
$$\vec{a} \times \left(\vec{a} \times (\vec{b} - \vec{c})\right) = \vec{a} \times \vec{0} = \vec{0}.$$

But

$$\vec{a} \times \left(\vec{a} \times (\vec{b} - \vec{c})\right) = \vec{a}\left(\vec{a} \cdot (\vec{b} - \vec{c})\right) - (\vec{b} - \vec{c})(\vec{a} \cdot \vec{a}) = \vec{a}(\vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}) - (\vec{b} - \vec{c})(\vec{a} \cdot \vec{a})$$
$$= \vec{a}(0) - (\vec{b} - \vec{c})(\vec{a} \cdot \vec{a}) = -(\vec{b} - \vec{c})(\vec{a} \cdot \vec{a}) = \vec{0}.$$

$\vec{a}$ is non-zero vector, so

$$(\vec{a} \cdot \vec{a}) \neq 0.$$

That's why

$$(\vec{b} - \vec{c}) = \vec{0} \ \ and \ \ \vec{b} = \vec{c}.$$