

Answer on Question #44081 – Math – Vector Calculus

Show that $(a - d) \times (b - c) + (b - d) \times (c - a) + (c - d) \times (a - b)$ is independent of d .

Solution.

$$(a - d) \times (b - c) = (a \times b) - (a \times c) - (d \times b) + (d \times c);$$

$$(b - d) \times (c - a) = (b \times c) - (b \times a) - (d \times c) + (d \times a);$$

$$(c - d) \times (a - b) = (c \times a) - (c \times b) - (d \times a) + (d \times b);$$

As we know for every vectors a and b the following is correct $(a \times b) = - (b \times a)$. Thus,

$$\begin{aligned} & (a - d) \times (b - c) + (b - d) \times (c - a) + (c - d) \times (a - b) = \\ &= (a \times b) - (a \times c) - (d \times b) + (d \times c) + (b \times c) - (b \times a) - (d \times c) + (d \times a) + (c \\ & \quad \times a) - (c \times b) - (d \times a) + (d \times b) = \\ &= (a \times b) - (a \times c) + (b \times d) - (c \times d) + (b \times c) + (a \times b) + (c \times d) - (a \times d) - (a \\ & \quad \times c) + (b \times c) + (a \times d) - (b \times d) = \\ &= 2(a \times b) - 2(a \times c) + 2(b \times c); \end{aligned}$$

As we can see the expression above is independent of d .