

Answer on Question #44080, Math, Vector Calculus

If $\vec{a}=(0,1,-1)$ and $\vec{c}=(1,1,1)$ are given vectors, then find a vector \vec{b} satisfying $\vec{a} \times \vec{b} + \vec{c} = 0$ and $\vec{a} \cdot \vec{b} = 3$.

Solution.

Assume that vector \vec{b} has coordinates (x,y,z) . Then $\vec{a} \times \vec{b} + \vec{c} = \begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ x & y & z \end{vmatrix} + (1,1,1) =$

$$i(z+y) - jx - kz + (1,1,1) = (1+y+z, 1-x, 1-x) = (0,0,0).$$

$$\text{Also, } \vec{a} \cdot \vec{b} = 0 \cdot x + 1 \cdot y - 1 \cdot z = y - z = 3.$$

Now we solve the system of equations:

$$\begin{cases} 1+y+z=0, \\ 1-x=0, \\ y-z=3; \end{cases} \begin{cases} 1+(z+3)+z=0, \\ x=1, \\ y=z+3; \end{cases} \begin{cases} z=-2, \\ x=1, \\ y=1. \end{cases}$$

Hence, $\vec{b} = (1,1,-2)$.

Answer: $(1,1,-2)$.