

Answer on Question #44079 – Math - Vector Calculus

Let $\vec{a} = (3, -1, 0)$ and $\vec{b} = (1/2, 3/2, 1)$. Find the vector $\vec{c}$ satisfying $\vec{a} \times \vec{c} = 4\vec{b}$ and $\vec{a} \cdot \vec{c} = 1$.

Solution

$$(\vec{a} \times \vec{c}) = 4\vec{b}$$

$$\vec{a} \times (\vec{a} \times \vec{c}) = 4\vec{a} \times \vec{b}.$$

Thus

$$\vec{a} \times (\vec{a} \times \vec{c}) = \vec{a}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{a}) = 4\vec{a} \times \vec{b}.$$

But $(\vec{a} \cdot \vec{c}) = 1$. So

$$\vec{c} = \frac{\vec{a} - 4\vec{a} \times \vec{b}}{(\vec{a} \cdot \vec{a})}.$$

$$(\vec{a} \cdot \vec{a}) = 3^2 + (-1)^2 + 0^2 = 10.$$

$$\vec{a} \times \vec{b} = \left((-1) \cdot 1 - 0 \cdot \frac{3}{2}; 0 \cdot \frac{1}{2} - 3 \cdot 1; 3 \cdot \frac{3}{2} - (-1) \cdot \frac{1}{2} \right) = (-1, -3, 5).$$

$$\vec{c} = \frac{(3, -1, 0) - 4(-1, -3, 5)}{10} = \left(\frac{7}{10}, \frac{11}{10}, -2 \right).$$

Answer: $\left(\frac{7}{10}, \frac{11}{10}, -2 \right)$.