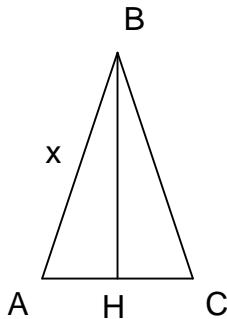


Question #44041 – Math – Geometry

The perimeter of an isosceles triangle is 42 cm. Its base is $2/3$ times the sum of equal sides. Find the length of each side and the area of the triangle.

Solution:



Let x be the length of equal sides ($AB = CB = x$), then $AC = 2x \cdot \frac{2}{3} = \frac{4x}{3}$ is the length of a base.

The perimeter of an isosceles triangle is:

$$P = AB + CB + AC = 2x + \frac{4x}{3}$$

Given $P = 42$.

$$2x + \frac{4x}{3} = 42$$

$$\frac{10x}{3} = 42$$

$$AB = CB = x = 12.6 \text{ cm}$$

$$AC = \frac{4x}{3} = \frac{4 \cdot 12.6}{3} = 16.8 \text{ cm}$$

Now given a triangle with sides $AB = CB = 12.6 \text{ cm}$ and $AC = 16.8 \text{ cm}$.

The formula for the area is: $A = \sqrt{p(p - AB)(p - CB)(p - AC)}$

$$\text{Where } p = \frac{P}{2} = \frac{42}{2} = 21$$

$$A = \sqrt{21(21 - 12.6)(21 - 12.6)(21 - 16.8)} \approx 79 \text{ cm}^2$$

Second method:

If BH is the altitude of the triangle, then the area of the triangle is:

$$A = \frac{AC \cdot BH}{2}$$

Using Pythagoras' theorem:

$$BH^2 + AH^2 = AB^2$$

If ΔABC is an isosceles triangle ($AB = CB$), then:

$$AH = HC = \frac{AC}{2} = \frac{16.8}{2} = 8.4 \text{ cm}$$

$$BH = \sqrt{AB^2 - AH^2} = \sqrt{12.6^2 - 8.4^2} \approx 9.4 \text{ cm}$$

$$A = \frac{16.8 \cdot 9.4}{2} \approx 79 \text{ cm}^2$$