Answer on Question #44039, Math, Algebra

if xyz = 1,then prove that 1/xyz^-1 + 1/xzy^-1 + 1/ yzx^-1 =1

Remark.

The statement of the problem is incorrect. For example when $x = y = z = 1$, then $xyz = 1$ and each of the summands in the left side of the equation

1/xyz^-1 + 1/xzy^-1 + 1/ yzx^-1 =1

equals 1. Hence

1/xyz^-1 + 1/xzy^-1 + 1/ yzx^-1 = 3.

We may prove that if $xyz = 1$ for some real x, y, z, then

$$\frac{1}{xy}z^{-1} + \frac{1}{xz}y^{-1} + \frac{1}{zy}x^{-1} = 3$$

and

$$\frac{1}{1 + x + y^{-1}} + \frac{1}{1 + y + z^{-1}} + \frac{1}{1 + z + x^{-1}} = 1.$$

Solution:

1) If $xyz = 1$, then

$$\frac{1}{xy}z^{-1} + \frac{1}{xz}y^{-1} + \frac{1}{zy}x^{-1} = 3.$$

The equality is true, as

$$\frac{1}{xy}z^{-1} + \frac{1}{xz}y^{-1} + \frac{1}{zy}x^{-1} = \frac{1}{xyz} + \frac{1}{xyz} + \frac{1}{xyz} = 1 + 1 + 1 = 3.$$

2) If $xyz = 1$, then

$$\frac{1}{1 + x + y^{-1}} + \frac{1}{1 + y + z^{-1}} + \frac{1}{1 + z + x^{-1}} = 1.$$

The equality is true, as if $xyz = 1$, then there exist real a, b, c such that $x = \frac{a}{b}$, $y = \frac{b}{c}$, $z = \frac{c}{a}$ (for example $a = xy$, $b = y$, $c = 1 = xyz$). Therefore

$$\frac{1}{1 + x + y^{-1}} + \frac{1}{1 + y + z^{-1}} + \frac{1}{1 + z + x^{-1}} = \frac{1}{1 + \frac{a}{b} + \frac{c}{b}} + \frac{1}{1 + \frac{b}{c} + \frac{a}{c}} + \frac{1}{1 + \frac{c}{a} + \frac{b}{a}}$$

$$= \frac{b}{a + b + c} + \frac{c}{a + b + c} + \frac{a}{a + b + c} = 1.$$