Problem. A height of a shotput can be modelled by the function
H = -4.9t2 + 8t + 1.5
where H is the height in metres and t is the time in seconds.

1) At what point do you think the shotput was traveling the fastest? What factors did you use to make your inference?
2) Determine the average rate of change on a short interval near the point you chose in question 1.
3) Estimate the instantaneous rate of change at the point you chose in question 1
4) Were your answers to the average rate of change the same as the instantaneous rate of change, if not why not?

Remark. We suppose that a shot put can be modeled by the function
$$H(t) = -4.9t^2 + 8t + 1.5.$$

Solution.
1) A shotput was traveling the fastest at the start (when $t = 0$) or when it reached ground ($H = 0$), as shotput slows down while it moves to the highest point and speeds up while it moves from the highest point to the ground. $H(0) = 1.5$ m, so a shotput was traveling the fastest when it reached ground (the distance from the start to the highest point is smaller than distance from the highest point to the point where shotput reached ground).
$H(T) = 0: -4.9T^2 + 8T + 1.5 = 0$ (T time when a shotput reached ground).
$\sqrt{D} = \sqrt{8^2 + 4 \cdot 4.9 \cdot 1.5} \approx 9.66$.
Then $T \approx \frac{-8-9.66}{-9.8}$ or $T \approx \frac{-8+9.66}{-9.8}$.
Therefore $T \approx 1.8\ s$, as T is positive.
2) We will determine it at interval $(1.79, 1.81)$
$H(1.79) = 0.11991$ m, $H(1.81) = -0.07289$ m.
$$\frac{\Delta H}{\Delta t} = \frac{H(1.81) - H(1.79)}{1.81 - 1.79} = \frac{-0.07289 - 0.11991}{1.81 - 1.79} = -9.64 \text{ m/s}.$$
3) $v(1.8) = -9.66$ m/s.
$H(1.79) = 0.11991$ m, $H(1.81) = -0.07289$ m.
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4) They aren't the same, as average rate of change is the rate of change over the interval, whereas the instantaneous rate of change is the rate of change at the specific point. The instantaneous rate of change could be approximates with average rate of change.