Answer on Question#43899 – Math – Linear Algebra

Question. Determine if the set $W = \{x: (x_1, x_2)\ such\ that\ x_1 = -x_2\}$ is a subspace of $\mathbb{R}^2$ or not.

Solution. We can rewrite the set W in the next form: $W = \{(y, -y)\} \in \mathbb{R}^2$. To determine if the set W is a subspace of $\mathbb{R}^2$ or not we shall use the next criterion of subspace: the subset W of linear space V is a subspace of $V \Leftrightarrow \begin{cases} (\bar{a} + \bar{b}) \in W\ \forall \bar{a}, \bar{b} \in W \\ \lambda\bar{a} \in W\ \forall \lambda \in \mathbb{R}, \forall \bar{a} \in W \end{cases}$.

Let $\bar{a} = (y_1, -y_1) \in W, \bar{b} = (y_2, -y_2) \in W$. Then

$\bar{a} + \bar{b} = (y_1 + y_2, -y_1 - y_2) = \big(y_1 + y_2, -(y_1 + y_2)\big)$. Obviously $(\bar{a} + \bar{b}) \in W$.

$\lambda\bar{a} = (\lambda y_1, -\lambda y_1) \in W\ \forall \lambda \in \mathbb{R}$.

Since $\begin{cases} (\bar{a} + \bar{b}) \in W\ \forall \bar{a}, \bar{b} \in W \\ \lambda\bar{a} \in W\ \forall \lambda \in \mathbb{R}, \forall \bar{a} \in W \end{cases}$ the set W is a subspace of $\mathbb{R}^2$.

Answer: The set $W = \{x: (x_1, x_2)\ such\ that\ x_1 = -x_2\}$ is a subspace of $\mathbb{R}^2$.