Answer on Question #43845 – Math - Statistics and Probability

A study was conducted to determine whether an expectant mother's cigarette smoking has any effect on the bone mineral content of her otherwise healthy child. A sample of 77 newborns whose mothers smoked during pregnancy has mean bone mineral content x-bar1 = 0.098 g/cm and standard deviation s1 = 0.026 g/cm; a sample of 161 infants whose mothers did not smoke has mean x-bar2 = 0.095 g/cm and standard deviation s2 = 0.025 g/cm. Assume that the underlying population variances are equal.

a. Are the two samples paired or independent?

b. State the null and alternative hypotheses of the two-sided test.

c. Conduct the test at the 0.05 level of significance. What do you conclude?

Solution

a. Clearly, the two samples are mutually exclusive; that is, an infant cannot have been born to a mother which smoked and did not smoke. So, each of these samples must have been chosen independently of each other. Another indicator that hints at independent samples is the size of the samples. To have samples paired, we require a one-to-one and onto (bijective) correspondence between two samples. This is not the case here. Since our sample sizes are $n_1 = 77$ and $n_2 = 161$, we are immediately led to believe that <u>the samples are independent of each other.</u>

b. For $i = 1,2, \dots, 77$, let X_{1_i} be the measured bone density of the i^{th} newborn to a mother who smoked. Accordingly, for $j = 1,2, \dots, 161$, let X_{2_j} be the measured bone density of the j^{th} newborn to a mother who did not smoke. We assume each of the X_{1_i} are i.i.d. (independently identically distributed) normal random variables with mean $\mu_{X_1}$and variance $\sigma^2_{X_1}$, and that each of the X_{2_j} are i.i.d. normal random variables with mean $\mu_{X_2}$and variance $\sigma^2_{X_2}$. We also assume that each of the samples is independent of each other. Then, our two-sided hypothesis test is
$H_0: \mu_{X_1} = \mu_{X_2}$
$H_1: \mu_{X_1} \neq \mu_{X_2}$.

c. Since the two sample variances are assumed to be equal, and that the underlying distribution of bone mineral content in infants is normally distributed (our samples are large enough in this case, that the normal assumption does not necessarily need to hold), our test statistic is

$$t = \frac{\overline{X_1} - \overline{X_2} - (\mu_{X_{1_0}} - \mu_{X_{2_0}})}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

where $\mu_{X_{1_0}}\ and\ \mu_{X_{2_0}}$ are the values of $\mu_{X_1}\ and\ \mu_{X_2}$ under the assumption that the null hypothesis is true, n_1 and n_2 are the respective sample sizes of X_1 and X_2, and

$$S_p^2 = \frac{(n_1 - 1)S^2_{X_1} + (n_2 - 1)S^2_{X_2}}{n_1 + n_2 - 2}.$$

We find that $S_p^2 \approx 0.0006414 \left(\frac{g}{cm}\right)^2$, so that $S_p = 0.025326 \frac{g}{cm}$. So, then our test statistic is

$t = 0.854918025$. Our t-table doesn't provide the values for $236(n_1 + n_2 - 2)$ degrees of freedom. However, $t_{236,0.025} \approx z_{0.025} = 1.96$. Since we have two-sided test, our p-value is

$p = 2 \cdot P(T \geq 0.854918025)$, where $T \sim T_{236}$. Looking at the z-table for $z = 0.85$, we obtain that $P(T \geq 0.854918025) \approx 0.197662543$, so that our p-value, $p > 0.38$. Since $p > 0.05$, we accept null hypothesis.

The data presented show that the population mean bone content in infants ($\frac{g}{cm}$) whose mothers smoked during pregnancy is not statistically significantly different from the population mean bone content in infants ($\frac{g}{cm}$) whose mothers did not smoke during pregnancy ($p > 0.38$).