Answer on Question #43816 – Math – Linear Algebra

Show that the vectors (1-i,i) and (2,-I+i) in R^2 are Linearly Dependent over Field but Linearly Independent over R , where i = √-1

Solution.

There are several inaccuracies in the condition. The first one vectors (1-i,i) and (2,-I+i) do not belong to R^2 because their coordinates do not belong to R, so R^2 should be changed on C^2 and there missed a field over which these vectors are linearly dependent, we guess there would have to be C.

Two vectors v_1, v_2 are Linearly Dependent over field F if there exist scalar a and b () in F such that

$$av_1 + bv_2 = 0$$

Let's try to find d these scalars

$$a(1-i, i) + b(2, -1+i) = 0$$

We get a system:

$$\begin{cases} a(1-i) + 2b = 0 \\ ai + b(-1+i) = 0 \end{cases} \qquad \begin{cases} b = \frac{a}{2}(-1+i) \\ ai + \frac{a}{2}(-1+i)(-1+i) = 0 \end{cases}$$

$$ai + \frac{a}{2}(-1+i)(-1+i) = 0$$

$$ai + \frac{a}{2}(1 - 2i - 1) = 0$$

$$a - a = 0$$

So, we get that a can be arbitrary and $b = \frac{a}{2}(-1+i)$. Indeed, if take a=2 then b =(-1+i).

And it is easy to check that $2(1-i, i) + (-1+i)(2, -1+i) = 0$, since (-1+i) ∈ C, we get that these vectors are linearly dependent over C. And as $b = \frac{a}{2}(-1+i)$ we see that a and b can not be real simultaneously, so these vectors are not linearly dependent over R, hence they are linearly independent over R.