Answer on Question #43815 – Math – Linear Algebra

Let A be an n×n matrix. Then Show that the set, U={u ϵ Rn : Au = - 3un} is a Subspace of Rn.

Solution.

Let V be a vector space over field K. Suppose that W is a subset of V. If W is a vector space itself (which means that it is closed under operations of addition and scalar multiplication), with the same vector space operations as V has, then W is a subspace of V. Then W is a subspace of V if and only if W satisfies the following condition:

If x ∈ W and y ∈ W then ax+by ∈ W for all a, b ∈ K.

In our case $V = R^n$, W=U and K=R. Let's verify the previous condition

Let x ∈ U and y ∈ U then Ax=-3nx and Ay=-3ny.

Let's consider the combination u=ax+by

Au=A(ax+by)=aAx+bAy=a(-3nx)+b(-3ny)=-3(ax+by)n=-3un, so we get

Au=-3un, hence U is a subspace of R^n.