Question 1
A student is taking a multiple-choice exam in which each question has four choices. Assume that the student has no knowledge of the correct answers to any of the questions. She has decided on a strategy in which she will place 4 balls (marked A, B, C and D) into a box. She randomly selects one ball for each question and replaces the ball in the box. The marking on the ball will determine her answer to the question. There are five multiple-choice questions on the exam. What is the probability that she will get
a. Five questions correct
b. At least four questions correct
c. No questions correct
d. No more than two questions correct
Solution:
The probability that student gives the correct answer equals $\frac{1}{4}$. The probability that student gives incorrect answer equals $q = 1 - \frac{3}{4}$. By the Bernoulli trial formula the probability that exactly k question are correct equals

$$P_k \quad \binom{5}{k} p^k q^{5-k} \quad \binom{5}{k} \left(\frac{1}{4}\right)^k \left(\frac{3}{4}\right)^{5-k}.$$

   a. The probability equals

$$P_a \quad P_5 \quad \binom{5}{5}\left(\frac{1}{4}\right)^5 \quad 0.0009765625.$$

   b. The probability equals

$$P_b \quad P_4 + P_5 \quad \binom{5}{4}\left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right) + P_5 \quad 0.0205078125.$$

   c. The probability equals

$$P_c \quad P_0 \quad \binom{5}{0}\left(\frac{3}{4}\right)^5 \quad 0.2373046875.$$

   d. The probability equals

$$P_d \quad P_0 + P_1 + P_2 \quad P_0 + \binom{5}{1}\left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^4 + \binom{5}{2}\left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^3 \quad 0.896484375.$$

Answer: $P_a \quad 0.0009765625, P_b \quad P_4 \quad 0.0205078125, P_c \quad 0.2373046875, P_d$ 0.896484375.

Question 2
A student majoring in accounting is trying to decide on the number of firms to which he should apply. Given his work experience and grades, he can expect to receive a job offer from 70% of the firms to which he applies. The student decides to apply to only four firms. What is the probability that he receives no job offers?
Solution:
The probability that he receives job offer from some firm equals $p = 0.7$. The probability that he doesn't receive job offer from some firm equals $q = 1 - p = 0.3$. By the Bernoulli trial formula the probability equals

$$P \quad \binom{4}{0} \quad ^0 q^4 \quad (0.3)^4 \quad 0.0081.$$

Answer: $P \quad 0.0081.$