Answer on Question #43760 – Math – Algebra

Question:

Solve for x: $\sqrt{(2x+9)(x+3)} - \sqrt{x^2 - x - 12} = 3\sqrt{x+3}$

Solution. Since

$$x^2 - x - 12 = (x+3)(x-4)$$

Then our equation can be rewritten in the following way

$$\sqrt{(2x+9)(x+3)} - \sqrt{(x+3)(x-4)} = 3\sqrt{x+3}$$

$$\sqrt{(2x+9)(x+3)} - \sqrt{(x+3)(x-4)} - 3\sqrt{x+3} = 0$$

$$\sqrt{x+3}(\sqrt{2x+9} - \sqrt{x-4} - 3) = 0$$

So we get $\sqrt{x+3} = 0, x_1 = -3$

or

$$\sqrt{2x+9} - \sqrt{x-4} - 3 = 0$$

$$\sqrt{2x+9} = \sqrt{x-4} + 3$$

$$2x + 9 = x - 4 + 6\sqrt{x-4} + 9$$

$$x + 4 = 6\sqrt{x-4}$$

$$x^2 + 8x + 16 = 36(x-4)$$

$$x^2 - 28x + 160 = 0$$

Using Vieta's theorem, we obtain $x_2 = 8, x_3 = 20.$ All the roots satisfy the initial equation.

Answer. $x_1 = -3, x_2 = 8, x_3 = 20.$