

Answer on Question #43732 – Math – Trigonometry

Prove that $4 \sin^3 a \cdot \cos 3a + 4 \cos^3 a \cdot \sin 3a = 3 \sin 4a$.

Solution.

To prove identity, we will transform its left part. For this, we can use next formulas:

$$\sin 3a = 3 \sin a - 4 \sin^3 a,$$

$$\cos 3a = 4 \cos^3 a - 3 \cos a.$$

Expressing from these formulas $4 \sin^3 a$ and $4 \cos^3 a$, and substituting it in the left part of our identity, we obtain:

$$\begin{aligned} (3 \sin a - \sin 3a) \cos 3a + (\cos 3a + 3 \cos a) \sin 3a &= 3 \sin a \cdot \cos 3a - \sin 3a \cdot \\ &\cdot \cos 3a + \cos 3a \cdot \sin 3a + 3 \cos a \cdot \sin 3a; \end{aligned}$$

We see that the second and the third members canceled. To end this proving we'll use next formula:

$$\sin(a + b) = \sin a \cdot \cos b + \sin b \cdot \cos a.$$

Thus, we have:

$$3 \sin(a + 3a) = 3 \sin 4a.$$